

1. FACTOR THEOREM

You can be asked to prove the **Factor Theorem** in Questions 2 and 3 (Algebra).

STATEMENT OF THE FACTOR THEOREM (FOR A CUBIC)

If $f(x) = ax^3 + bx^2 + cx + d$ and if $f(k) = 0$ then $(x - k)$ is a factor of $f(x)$ and vice versa.

PROOF

$$f(x) = ax^3 + bx^2 + cx + d$$

$$f(k) = ak^3 + bk^2 + ck + d$$

$$\therefore f(x) - f(k) = a(x^3 - k^3) + b(x^2 - k^2) + c(x - k)$$

$$= (x - k)\{ax^2 + akx + ak^2 + bx + bk + c\}$$

$$= (x - k)g(x)$$

$$\therefore f(x) = f(k) + (x - k)g(x)$$

$$(i) f(k) = 0 \Rightarrow f(x) = (x - k)g(x)$$

$$\therefore x - k \text{ is a factor.}$$

$$(ii) x - k \text{ is a factor} \Rightarrow f(k) = 0$$

Can you prove the Factor Theorem for a quadratic:

$$f(x) = ax^2 + bx + c?$$

2. DIFFERENTIATION RULES

You can be asked to prove certain **differentiation rules** in Questions 6 and 7 (Differentiation).

2.1 THE SUM RULE

STATEMENT OF THE SUM RULE

$$\text{If } y = u + v \text{ then } \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

PROOF

$$\begin{aligned} y &= u + v \\ y + \Delta y &= (u + \Delta u) + (v + \Delta v) \\ y &= u + v \end{aligned}$$

$$\Delta y = \Delta u + \Delta v$$

$$\therefore \frac{\Delta y}{\Delta x} = \frac{\Delta u}{\Delta x} + \frac{\Delta v}{\Delta x}$$

$$\therefore \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

2.2 THE PRODUCT RULE

STATEMENT OF THE PRODUCT RULE

$$\text{If } y = u.v \text{ then } \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

PROOF

$$\begin{aligned} y &= uv \\ y + \Delta y &= (u + \Delta u)(v + \Delta v) = uv + u\Delta v + v\Delta u + \Delta u.\Delta v \\ y &= uv \end{aligned}$$

$$\Delta y = u\Delta v + v\Delta u + \Delta u.\Delta v$$

$$\therefore \frac{\Delta y}{\Delta x} = u \frac{\Delta v}{\Delta x} + v \frac{\Delta u}{\Delta x} + \frac{\Delta u}{\Delta x} \Delta v$$

$$\therefore \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$