

# 1. COUNTING TECHNIQUES

## INTRODUCTION

If there are 11 teams in a football competition and each team is to play the other only once, how many fixtures must be organised? How many poker hands will be all hearts? Such questions are very tricky and so require a few nifty tricks. We will deal with three techniques:

### 1.1 The Multiplication Principle (MP)

#### 1.2 Arrangements

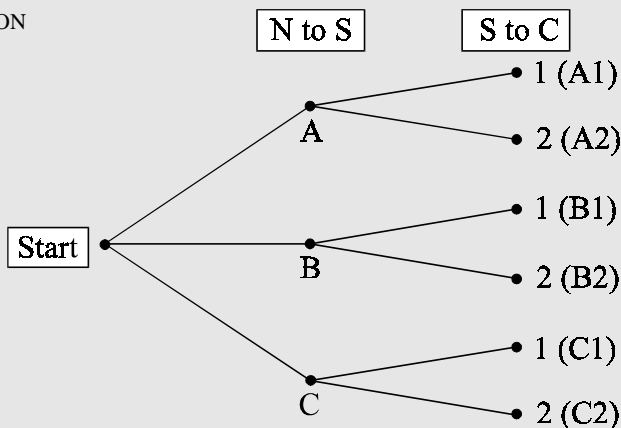
#### 1.3 Selections

### 1.1 THE MULTIPLICATION PRINCIPLE

**THE IDEA:** This is the most important technique in counting techniques and is sometimes known as the “AND THEN” rule.

**Example 1:** If there are 3 escape routes from the NORTHSIDE of a city to the SOUTHSIDE and 2 routes to escape from the SOUTHSIDE to the COUNTRY, how many ways can you escape from the NORTHSIDE to the SOUTHSIDE **and then** from the SOUTHSIDE to the COUNTRY?

SOLUTION



Therefore, there are 6 ( $= 3 \times 2$ ) routes of escaping from the NORTHSIDE to the SOUTHSIDE **and then** from the SOUTHSIDE to the COUNTRY.

### STATEMENT OF MP

If there are  $a$  ways of performing one operation and  $b$  ways of performing a second operation and  $c$  ways of performing a third operation, then the number of ways of performing the first operation **and then** the second operation **and then** the third operation is given by  $a \otimes b \otimes c \otimes \dots$ .

The tree diagram in the previous example is very cumbersome and so we will use boxes in future.

**Example 2:** A restaurant has 3 starters, 5 main courses and 4 desserts. How many ways can you have a 3 course meal?

SOLUTION



$$\text{Number of ways} = 3 \times 5 \times 4 = 60$$

**Example 3:** A bicycle lock has 2 letter wheels and 3 digit wheels. How many possible opening possibilities are there?

SOLUTION



$$\text{Number of ways} = 26 \times 26 \times 10 \times 10 \times 10 = 676,000$$

### MP TRICKS

**TRICK 1. The MP can be used to arrange objects in order.**

**Example 4:** How many ways can the letters of the word SWORD be arranged if no letters can be repeated?

SOLUTION

5 letters SWORD



$$\text{Number of ways} = 5 \times 4 \times 3 \times 2 \times 1 = 120, \text{ i.e. all orders}$$

**EXPLANATION:** There are 5 ways to fill the first box. Once it is filled, there are 4 letters left in which to fill the second box. Once the first 2 boxes are filled there are 3 letters left to fill the third box and so on.

**TRICK 2. Conditions or restrictions should be built in at the start if possible.**

**Example 5:** How many 4 digit numbers end in 7?

SOLUTION

—Restriction built in

Number of ways =  $9 \times 10 \times 10 \times 1 = 900$

You cannot have a 0, otherwise it would be a 3 digit number.

**TRICK 3. Dependence:** Sometimes a choice depends on what happened before. This means you have to take account of restrictions as you go along.

**Example 6:** How many 4 digit numbers end in 7 if no digits can be repeated?

SOLUTION

You cannot have a 0 or 7

—Restriction built in first

Number of ways =  $8 \times 8 \times 7 \times 1 = 448$

You cannot have a 7 or what's in Box 1, but you can have 0.

**Example 7:** How many 4 digit numbers are there that are divisible by 5 if there are no repeated digits?

SOLUTION

Divisible by 5 means that the last digit is 0 OR 5.

(i) End in 0     —Restriction built in first

Number of ways =  $9 \times 8 \times 7 \times 1 = 504$

OR (ii) End in 5     —Restriction built in first

Number of ways =  $8 \times 8 \times 7 \times 1 = 448$

**TRICK:** OR  $\Rightarrow \oplus 504 + 448 = 952$