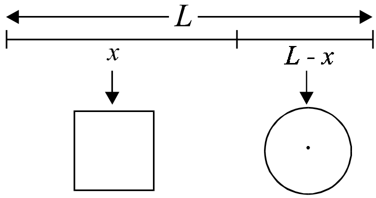
2.2 SNIPPERS

This involves cutting a piece of wire (string) and reforming it into shapes.

SNIPPER TRICKS

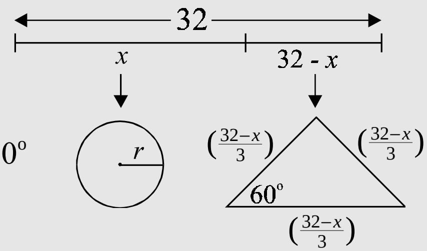
1. If the perimeter of one shape is x the perimeter of the other is $(L - x)$.



2. The sum of the perimeters is equal to the length of the string.

Example 6: A wire of length 32 m is cut into 2 pieces. One piece is bent to form a circle of radius r and the other into an equilateral triangle. Express the sum of the areas in terms of r. Find r if the sum of the areas is a minimum.

SOLUTION



1. S (Sum of areas)
2. Draw a diagram.
3. $S = \pi r^2 + \frac{1}{2}\left(\frac{32-x}{3}\right)\left(\frac{32-x}{3}\right)\sin 60^{\circ}$

$\therefore S = \pi r^2 + \frac{(32-x)^2\sqrt{3}}{36}$

4. Perimeter of Circle $= 2\pi r = x$ (Extra information)
5. $S = \pi r^2 + \frac{(32-2\pi r)^2\sqrt{3}}{36}$
6. $\frac{dS}{dr} = 2\pi r + \frac{\sqrt{3}}{36}[2(32-2\pi r)(-2\pi)] = 0$

$\therefore 18r - \sqrt{3}(32 - 2\pi r) = 0$

$\therefore r = \frac{32\sqrt{3}}{18 + 2\sqrt{3}\pi} = \frac{16\sqrt{3}}{9 + \sqrt{3}\pi}$

2.3 INSCRIBERS

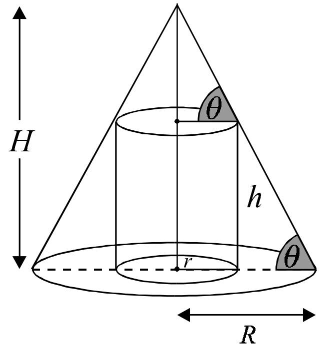
These are situations where one shape is put inside another.

INSCRIBER TRICKS

1. For a circle or a sphere always mark the centre and draw an intelligent radius.
2. Look out for Pythagoras.
3. A cylinder inside a cone.

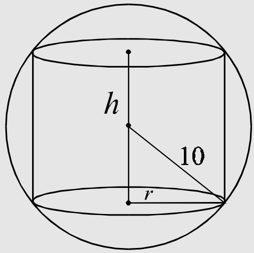
$\tan\theta = \frac{H}{R} = \frac{h}{R-r} = \frac{H-h}{r}$

4. Concentrate on the shape to be maximized/minimized.



Example 7: Find the height of the circular cylinder of maximum volume which can be inscribed in a sphere of radius 10 cm.

SOLUTION



1. V (Cylinder)
2. Draw a diagram.
3. $V = \pi r^2 h$
4. $r^2 + \frac{h^2}{4} = 100$ (Pythagoras)

$\therefore r^2 = 100 - \frac{h^2}{4}$

5. $V = \pi(100 - \frac{h^2}{4})h = 100\pi h - \frac{\pi h^3}{4}$
6. $\frac{dV}{dh} = 100\pi - \frac{3\pi h^2}{4} = 0 \Rightarrow 400 = 3h^2$

$\therefore \boxed{h = \frac{20}{\sqrt{3}}}$ $h = -\frac{20}{\sqrt{3}}$ (Reject)