3.2 INVERSE MATRIX

[A] **Definition**: The inverse A^{-1} of a 2×2 matrix A is a matrix such that $AA^{-1} = I = A^{-1}A$ where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

[B] **Finding the Inverse** of a 2×2 matrix, A:

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $A^{-1} = \frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ where $\Delta = (ad - bc)$.

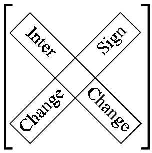
Note: $\Delta = (ad - bc)$ is called the **determinant** of a matrix.

If $\Delta = 0$ the inverse A^{-1} does not exist.

TRICK

Calculating the inverse of $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

Find $\Delta = (ad - bc)$,

then find $A^{-1} = \frac{1}{\Delta}$ 

Example 4: Find A^{-1} if $A = \begin{pmatrix} 2 & -1 \\ 3 & 7 \end{pmatrix}$ and show $AA^{-1} = I$.

SOLUTION

$$A = \begin{pmatrix} 2 & -1 \\ 3 & 7 \end{pmatrix}$$

$$\Delta = 14 - (-3) = 17$$

$$A^{-1} = \tfrac{1}{17}\begin{pmatrix} 7 & 1 \\ -3 & 2 \end{pmatrix}$$

$$AA^{-1} = \begin{pmatrix} 2 & -1 \\ 3 & 7 \end{pmatrix}\begin{pmatrix} 7 & 1 \\ -3 & 2 \end{pmatrix}\tfrac{1}{17} = \tfrac{1}{17}\begin{pmatrix} 17 & 0 \\ 0 & 17 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$