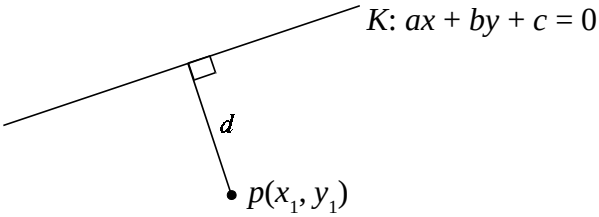
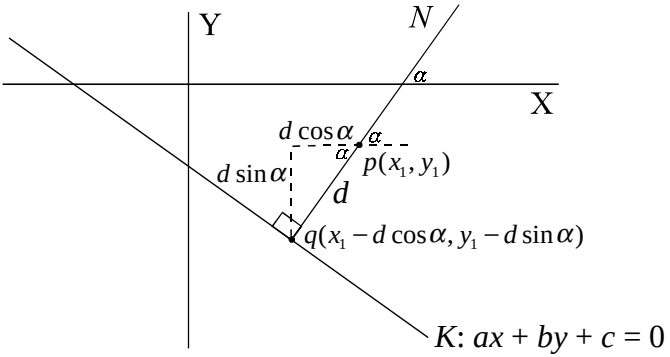
2.3 THE PERPENDICULAR DISTANCE FROM A POINT TO A LINE (PIL)

[A] **THE IDEA**: We want to find a formula for the perpendicular distance d.



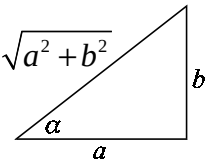
[B] **PROOF**



1. $q \in K \Rightarrow\ a(x_1 - d\cos\alpha) + b(y_1 - d\sin\alpha) + c = 0$

$\Rightarrow ax_1 + by_1 + c = d(a\cos\alpha + b\sin\alpha)$

2. Slope of $N = \tan\alpha = \frac{b}{a}$ since $N \perp K$



$$\therefore a\cos\alpha + b\sin\alpha = \frac{a.a}{\sqrt{a^2+b^2}} + \frac{b.b}{\sqrt{a^2+b^2}}$$

$$= \frac{a^2+b^2}{\sqrt{a^2+b^2}} = \sqrt{a^2+b^2}$$

3. $$\therefore d = \frac{|ax_1 + by_1 + c|}{a\cos\alpha + b\sin\alpha} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

REMEMBER IT AS: $d = \dfrac{|\text{PIL}|}{\sqrt{\text{PYL}}} = \dfrac{|\text{Point Into Line}|}{\sqrt{\text{PYthagoras on Line}}}$

NOTES

1. Mod on top to ensure d is positive.
2. It is very important as it is used a lot in the circle.

[C] **TYPES OF PROBLEM**

TYPE 1. TO FIND THE PERPENDICULAR DISTANCE FROM A POINT TO A LINE

Example 10: Find the perpendicular distance from $p(-2, -3)$ to the line K: $4x - 5y + 1 = 0$.

SOLUTION

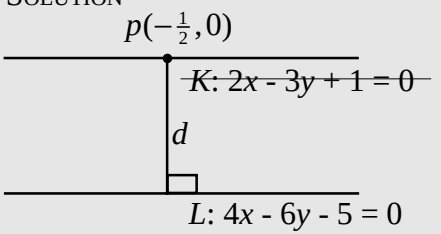
$p(-2, -3)$, K: $4x - 5y + 1 = 0$

$$d = \frac{|\text{PIL}|}{\sqrt{\text{PYL}}} = \frac{|4(-2) - 5(-3) + 1|}{\sqrt{4^2 + (-5)^2}} = \frac{|8|}{\sqrt{41}} = \frac{8}{\sqrt{41}} = \frac{8\sqrt{41}}{41}$$

TYPE 2. TO FIND THE PERPENDICULAR DISTANCE BETWEEN PARALLEL LINES

Example 11: Find the perpendicular distance between the parallel lines K: $2x - 3y + 1 = 0$ and L: $4x - 6y - 5 = 0$.

SOLUTION



STEPS

1. Choose a point p on one line.
2. Cross out this line.
3. PIL p with other line.

$$\therefore d = \frac{|4(-\tfrac{1}{2}) - 6(0) - 5|}{\sqrt{16 + 36}} = \frac{7}{\sqrt{52}} = \frac{7\sqrt{13}}{26}$$

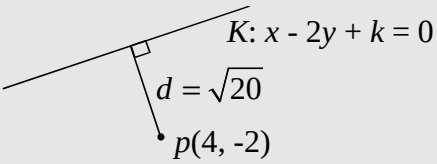
TRICK

If PIL = 0 the point is on the line.

TYPE 3. TO FIND THE EQUATIONS OF LINES OF A GIVEN SLOPE AT A CERTAIN DISTANCE FROM A FIXED POINT

Example 12: Find the equations of the lines of slope $\frac{1}{2}$ which are $\sqrt{20}$ units from (4, -2).

SOLUTION



$$d = \frac{|\text{PIL}|}{\sqrt{\text{PYL}}} = \sqrt{20}$$

$$\Rightarrow \frac{|4+4+k|}{\sqrt{1+4}} = \sqrt{20}$$

$$\Rightarrow |k+8| = 10 \Rightarrow k+8 = \pm 10$$

$$\therefore k = 2, -18$$

$\therefore$ K_1: x - 2y + 2 = 0

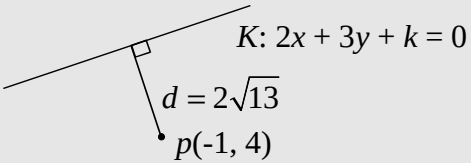
K_2: x - 2y - 18 = 0

STEPS

1. Draw a simple one line picture.
2. Write the line with slope $\frac{1}{2}$ as x - 2y + k = 0.
3. The PIL formula gives both answers because of modulus.

Example 13: Find the equations of the lines which are perpendicular to 3x - 2y + 1 = 0 and $2\sqrt{13}$ units from (-1, 4).

SOLUTION



$$d = \frac{|\text{PIL}|}{\sqrt{\text{PYL}}} = 2\sqrt{13}$$

$$\Rightarrow \frac{|-2+12+k|}{\sqrt{4+9}} = 2\sqrt{13}$$

$$\Rightarrow |k+10| = 26 \Rightarrow k+10 = \pm 26$$

$$\therefore k = 16, -36$$

$\therefore$ K_1: 2x + 3y + 16 = 0

K_2: 2x + 3y - 36 = 0

TRICK

2x + 3y + k = 0

$\perp$

3x - 2y + 1 = 0