1.2 ALGEBRAIC INFINITY LIMITS

$\lim_{x\to\infty} f(x)$ where $f(x)$ is algebraic

STEPS

1. Plonk in ∞ into the function. If it gives a non-IF answer, take it.

2. If you get $\frac{\infty}{\infty}$ then: (i) Take out the highest power of the variable from each bracket on the top.
(ii) Do the same for the bottom.
(iii) Tidy up the takeouts.
(iv) Plonk in ∞ for the variable.

Example 5: Find $\lim_{n\to\infty} \frac{5}{n^2+1}$.

SOLUTION

$$\lim_{n\to\infty} \frac{5}{n^2+1} = \frac{5}{\infty} = 0$$

Example 6: Find $\lim_{n\to\infty} \frac{3n^2-2n+1}{4n^2+5n-1}$.

SOLUTION

$$\lim_{n\to\infty} \frac{3n^2-2n+1}{4n^2+5n-1} = \lim_{n\to\infty} \frac{\cancel{n^2}(3-\frac{2}{n}+\frac{1}{n^2})}{\cancel{n^2}(4+\frac{5}{n}+\frac{1}{n^2})} = \frac{3-0+0}{4+0-0} = \frac{3}{4}$$

Example 7: Evaluate $\lim_{n\to\infty} \frac{(4n-1)(n^2+5)(2n^3+1)}{(3n^2-1)^3}$.

SOLUTION

$$\lim_{n\to\infty} \frac{(4n-1)(n^2+5)(2n^3+1)}{(3n^2-1)^3} = \lim_{n\to\infty} \frac{\cancel{n}(4-\frac{1}{n})\cancel{n^2}(1+\frac{5}{n^2})\cancel{n^3}(2+\frac{1}{n^3})}{\cancel{n^6}(3-\frac{1}{n^2})^3}$$

$$= \frac{(4)(1)(2)}{(3)^3} = \frac{8}{27}.$$

Example 8: If $S_n = \dfrac{n(n+1)(2n+1)}{3n(n-2)}$ find $\lim\limits_{n\to\infty} S_n$.

SOLUTION

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n(n+1)(2n+1)}{3n(n-2)} = \lim_{n\to\infty} \frac{\cancel{n}\,\cancel{n}(1+\frac{1}{n})n(2+\frac{1}{n})}{3\cancel{n}\,\cancel{n}(1-\frac{2}{n})}$$

$$= \frac{(1)(\infty)(2)}{3(1)} = \infty$$

Example 9: Evaluate $\lim\limits_{n\to\infty} u_n$ if $u_n = \dfrac{(8n^3+1)^{\frac{1}{3}}}{\sqrt{16n^2-5}}$.

SOLUTION

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{(8n^3+1)^{\frac{1}{3}}}{\sqrt{16n^2-5}} = \lim_{n\to\infty} \frac{\cancel{n}(8+\frac{1}{n^3})^{\frac{1}{3}}}{\cancel{n}(16-\frac{5}{n^2})^{\frac{1}{2}}} = \frac{8^{\frac{1}{3}}}{16^{\frac{1}{2}}} = \frac{2}{4} = \frac{1}{2}$$

1.3 TRIG LIMITS

$\lim\limits_{x\to a} f(x)$ where $f(x)$ is a trig function

STEPS

1. Simplify trig using tables.
2. Plonk in value of x (i.e. a) into the function.

(i) If it gives a non-IF answer, then take it.
(ii) If you get an IF then do HORIZONTAL SHIFT or L'Hopital.

Example 10: Evaluate $\lim\limits_{x\to 0} \dfrac{x}{\cos x}$.

SOLUTION

$$\lim_{x\to 0} \frac{x}{\cos x} = \frac{0}{1} = 0$$

TRICK

Horizontal shift depends on the famous limits:

$\lim_{\theta\to0}\frac{\sin\theta}{\theta}=1$ and $\lim_{\theta\to0}\frac{\theta}{\sin\theta}=1$ where angles **must be the same** and in radians.

Example 11: Evaluate $\lim_{x\to0}\frac{\sin 7x}{x}$.

SOLUTION

$$\lim_{x\to0}\frac{\sin 7x}{x}=\frac{\sin 0}{0}=\frac{0}{0}\ \textbf{(IF)}$$

HORIZONTAL SHIFT

$$\lim_{x\to0}\frac{\sin 7x}{x}=\lim_{x\to0}\left(\frac{\sin 7x}{7x}\right)\left(\frac{7\cancel{x}}{\cancel{x}}\right)$$

$$=1\times 7=7$$

L'HOPITAL'S RULE

$$\lim_{x\to0}\frac{\sin 7x}{x}=\lim_{x\to0}\frac{7\cos 7x}{1}$$

$$=7\cos 0=7$$

Example 12: Evaluate $\lim_{x\to0}\left(\frac{\sin 3x}{4x}+\frac{\sin^2 5x}{3x^2}\right)$.

SOLUTION

$$\lim_{x\to0}\left(\frac{\sin 3x}{4x}+\frac{\sin^2 5x}{3x^2}\right)=\frac{0}{0}+\frac{0}{0}\ \textbf{(IF)}$$

HORIZONTAL SHIFT

$$\lim_{x\to0}\left(\frac{\sin 3x}{4x}+\frac{\sin^2 5x}{3x^2}\right)$$

$$=\lim_{x\to0}\left(\frac{\sin 3x}{3x}\right)\left(\frac{3x}{4x}\right)+\lim_{x\to0}\left(\frac{\sin 5x}{5x}\right)\left(\frac{\sin 5x}{5x}\right)\left(\frac{25\cancel{x^2}}{3\cancel{x^2}}\right)$$

$$=1\times\tfrac{3}{4}+1\times1\times\tfrac{25}{3}=\tfrac{3}{4}+\tfrac{25}{3}=\tfrac{9+100}{12}=\tfrac{109}{12}$$