Example 5: Find Y-type asymptote of the curve $y=\dfrac{3}{x-2}$.

SOLUTION

$$y=\frac{3}{x-2}\Rightarrow y=\frac{3}{x(1-\frac{2}{x})}\Rightarrow \lim_{x\to\infty} y=\lim_{x\to\infty}\frac{3}{x(1-\frac{2}{x})}=\frac{3}{\infty(1)}=0$$

$\therefore \boxed{y=0}$ is a Y-type AS.

[C] Steps for plotting rational curves:

STEPS

1. Find asymptotes.
2. Build up a table by choosing 2 points to the left and 2 points to the right of the X-type AS.
3. **Free points**: Put $y = 0$, find x.
Put $x = 0$, find y.
4. Plot skimming along the asymptotes.

Example 6: Show that the curve $y=\dfrac{x+1}{x-2}$ has no LMax, LMin or PI's. Find the AS and plot it roughly. If the tangent at x_1 is parallel to the tangent at x_2 show $x_1+x_2=4$ ($x_1\neq x_2$).

SOLUTION

$$y=f(x)=\frac{x+1}{x-2}$$

$$\frac{dy}{dx}=\frac{(x-2)1-(x+1)1}{(x-2)^2}=\frac{-3}{(x-2)^2}=-3(x-2)^{-2}$$

Cont...

$$\frac{dy}{dx} = 0 \Rightarrow \frac{-3}{(x-2)^2} = 0 \Rightarrow -3 = 0$$

$\therefore$ no solutions, $\therefore$ no LMax or LMin

$$\frac{d^2y}{dx^2} = 6(x-2)^{-3} = \frac{6}{(x-2)^3} = 0 \Rightarrow 6 = 0$$

$\therefore$ no solutions, $\therefore$ no PI's

Step 1. AS (i) X-type: $x = 2$

(ii) Y-type: $y = \dfrac{x+1}{x-2} = \dfrac{\cancel{x}(1+\frac{1}{x})}{\cancel{x}(1-\frac{2}{x})}$

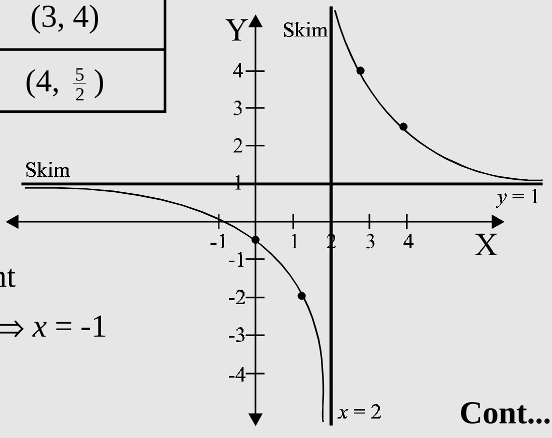
$\therefore x \to \infty, \Rightarrow y \to \frac{1}{1} = 1$

$\therefore y = 1$

Step 2. Draw up a table (x = 2 is the AS)

	x	y	Pt.
L	0	$-\frac{1}{2}$	$(0, -\frac{1}{2})$
	1	-2	(1, -2)
	2	AS	
R	3	4	(3, 4)
	4	$\frac{5}{2}$	$(4, \frac{5}{2})$

Step 4. Plot



Step 3. Free Point

Free point $y = 0 \Rightarrow x = -1$

Cont...