

[C] Differentiating Exponential (Expo) Functions

$$y = e^x \Rightarrow \frac{dy}{dx} = e^x$$

$$y = e^{f(x)} \Rightarrow \frac{dy}{dx} = e^{f(x)} \times f'(x)$$

TRICK

To diff an expofn, $y = e^{f(x)}$:
Repeat the whole function
× Diff of the power.

Example 17: Differentiate $y = \frac{e^{\frac{1}{x}}}{4}$.

SOLUTION

$$y = \frac{e^{\frac{1}{x}}}{4} = \frac{1}{4} e^{x^{-1}} \Rightarrow \frac{dy}{dx} = \frac{1}{4} e^{\frac{1}{x}} \left(-\frac{1}{x^2}\right) = -\frac{e^{\frac{1}{x}}}{4x^2}$$

Example 18: Differentiate $y = e^{2x^2}$.

SOLUTION

$$y = e^{2x^2} \Rightarrow \frac{dy}{dx} = e^{2x^2} \cdot 4x = 4xe^{2x^2}$$

Exponential Algebra

POWER RULES

1. $a^m \cdot a^n = a^{m+n}$

2. $\frac{a^m}{a^n} = a^{m-n}$

3. $a^0 = 1$

4. $a^{-n} = \frac{1}{a^n}$

5. $(a^m)^n = a^{mn}$

6. $(a^m b^n)^p = a^{mp} b^{np}$

7. $\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}$

8. $\left(\frac{a^m}{b^n}\right)^{-p} = \left(\frac{b^n}{a^m}\right)^p$

Note: $e \cong 2.7 > 0$

Example 19: If $y = \frac{(e^x)^2 e^{x^3-1}}{\sqrt{e^{x-3}}}$ find $\frac{dy}{dx}$.

SOLUTION

$$y = \frac{(e^x)^2 e^{x^3-1}}{\sqrt{e^{x-3}}} = \frac{e^{2x} e^{x^3-1}}{e^{\frac{1}{2}x-\frac{3}{2}}} = e^{x^3+\frac{3}{2}x+\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = e^{x^3+\frac{3}{2}x+\frac{1}{2}} (3x^2 + \frac{3}{2})$$

Example 20: If $y = \frac{x^2 e^x}{e^{3x}}$ find $\frac{dy}{dx}$.

SOLUTION

$$y = \frac{x^2 e^x}{e^{3x}} = x^2 \cdot e^{-2x}$$

Product rule: $\frac{dy}{dx} = x^2 \cdot e^{-2x} (-2) + e^{-2x} \cdot 2x = 2xe^{-2x} (-x+1)$

$$= \frac{(1-x)2x}{e^{2x}}$$

Example 21: Differentiate $y = \frac{e^{\cos^2 x}}{e^{\sin^2 x}}$.

SOLUTION

$$y = \frac{e^{\cos^2 x}}{e^{\sin^2 x}} = e^{\cos^2 x - \sin^2 x} = e^{\cos 2x}$$

$$\therefore \frac{dy}{dx} = e^{\cos 2x} \cdot -\sin 2x \cdot 2 = -2 \sin 2x e^{\cos 2x}$$