4.2 DE MOIVRE OBJECTS

The object is in the form ($\cos\theta + i\sin\theta$). We shall call it a De Moivre Object (**DMO**). There are 4 nifty tricks that we can do with it.

[A] **De Moivre Tricks**

1. **Trick**: If you multiply DMO's you just add the angles.

$$(\cos A \oplus i\sin A)(\cos B \oplus i\sin B)$$
$$= (\cos A\cos B - \sin A\sin B) + i(\cos A\sin B + \cos B\sin A)$$
$$= \cos(A+B) + i\sin(A+B)$$

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2. **Trick**: If you move a DMO from top to bottom or vice versa you change the sign between cos and sin.

This works for $\cos\theta \pm i\sin\theta$

$$\frac{1}{\cos A \pm i\sin A} = \frac{1}{\cos A \pm i\sin A}\,\frac{\cos A \mp i\sin A}{\cos A \mp i\sin A}$$
$$= \frac{\cos A \mp i\sin A}{\cos^2 A + \sin^2 A} = \cos A \mp i\sin A$$

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3. **Trick**: If you divide 2 DMO's you subtract their angles.

$$\frac{\cos A \oplus i\sin A}{\cos B \oplus i\sin B} = (\cos A + i\sin A)(\cos B - i\sin B)$$
$$= (\cos A\cos B + \sin A\sin B) + i(\sin A\cos B - \cos A\sin B)$$
$$= \cos(A-B) + i\sin(A-B)$$

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4. **Trick**: $(\cos A + i\sin A)^n$

$$= \underbrace{(\cos A + i\sin A)(\cos A + i\sin A)\ldots\ldots\ldots(\cos A + i\sin A)}_{n \text{ times}}$$

$(\cos nA + i\sin nA)$ by **Trick** 1. This is an example of De Moivre's Theorem (DMT).

A Summary of De Moivre's Tricks

DMO 1

If you multiply DMO's you just add the angles.

$$(\cos A \oplus i\sin A)(\cos B \oplus i\sin B) = \cos(A+B) + i\sin(A+B)$$

DMO 2

If you move a DMO from top to bottom or vice versa you change the sign between cos and sin.

$$\frac{1}{\cos A \pm i\sin A} = \cos A \mp i\sin A$$

DMO 3

If you divide 2 DMO's you subtract their angles.

$$\frac{\cos A \oplus i\sin A}{\cos B \oplus i\sin B} = \cos(A-B) + i\sin(A-B)$$

DMO 4

De Moivre's Theorem (DMT)

$$(\cos A + i\sin A)^n = (\cos nA + i\sin nA),\ n \in N_0$$

Trick: Don't be too hasty in changing complex numbers out of polar form. If you know the 4 DMO tricks it is usually easier to work in polars.

Example 4: Draw $3\left\{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right\}$ on an argand diagram.

Solution

$z = 3\left\{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right\}$

$r = |z| = 3,\ \theta = \frac{\pi}{3} = 60^{\circ}$