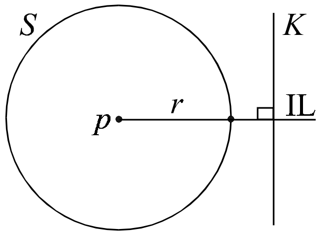
2. LINE AND CIRCLE

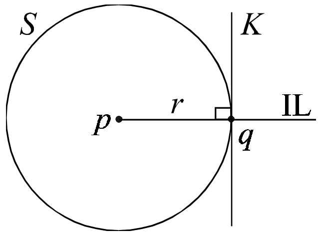
2.1 INTRODUCTION

When you have a line and a circle, 3 possibilities exist.

MISS	TOUCH

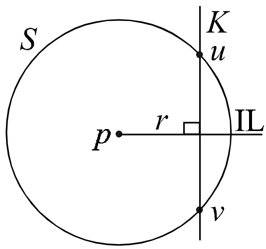


$S \cap K = \{\ \}$



$S \cap K = \{q\}$

CUT



$S \cap K = \{u, v\}$

TRICK

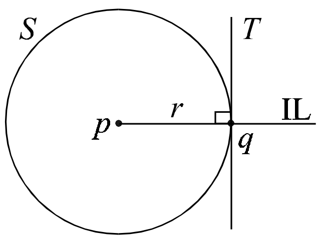
In every case of a line and a circle, always draw a line from the centre perpendicular to the line and call it the **IL** (intelligent line) - it's always a clever thing to do).

TANGENTS

1. The tangent, T, is perpendicular to the intelligent line, IL.
2. The perpendicular distance (PIL) from the centre to T equals the radius, r.
3. If you solve S and T you get a quadratic with only one solution,

i.e. $b^2 = 4ac$ in the Magic Formula.

4. $T \cap \text{IL} = \{q\}$. q is the point of contact.



Touch

2.2 FINDING THE EQUATION OF A TANGENT

3 types of problem arise.

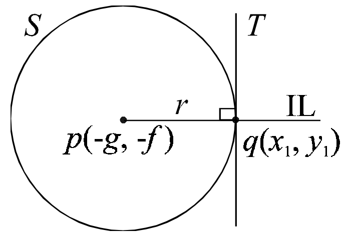
[A] TYPE 1: TANGENT AT

This means the point of contact is given. There are 2 methods:

METHOD 1

STEPS

1. Find the slope of IL from p and q.
2. Hence, find the slope of T since $T \perp$ IL.
3. Hence, find the equation of T since $q \in T$.



Example 1: Find the equation of the tangent to the circle $x^2 + y^2 - 8x + 10y + 1 = 0$ at (2, 1).

SOLUTION

Slope of pq = Slope of IL

$= -\frac{6}{2} = -3$

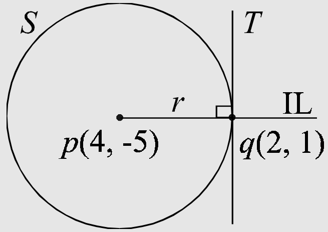
$\therefore$ Slope of $T = \frac{1}{3}$

$\therefore$ Equation T: $x - 3y + k = 0$

$(2, 1) \in T \Rightarrow 2 - 3 + k = 0$

$\therefore k = 1$

T: $x - 3y + 1 = 0$



$r = \sqrt{16 + 25 - 1} = \sqrt{40}$

METHOD 2

The equation of a circle is simply the equation of a curve so that the slope of the tangent can be found by calculating $\frac{dy}{dx}$.

Example 2: Find the equation of the tangent to the circle $x^2 + y^2 - 8x + 10y + 1 = 0$ at (2, 1).

TRICK

Find $\frac{dy}{dx}$ by differentiating implicitly wrt *x*.

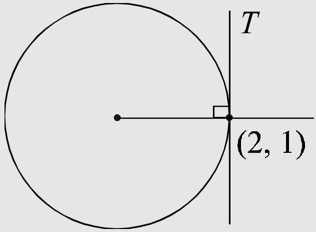
SOLUTION

$$\Rightarrow 2x + 2y\frac{dy}{dx} - 8 + 10\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx}(2y + 10) = 8 - 2x$$

$$\therefore \frac{dy}{dx} = \frac{4 - x}{y + 5}$$

$$\therefore \left(\frac{dy}{dx}\right)_{(2,1)} = \frac{2}{6} = \frac{1}{3}$$



Eqn *T*: $x - 3y + k = 0$

$(2, 1) \in T \Rightarrow 2 - 3 + k = 0 \therefore k = 1$

T: $x - 3y + 1 = 0$

[B] TYPE 2: TANGENTS WHOSE SLOPE IS GIVEN DIRECTLY OR INDIRECTLY

STEPS

1. Write down the equation of *T* in the form $ax + by + c = 0$.
2. [PIL centre with *T*] = *r*.