

$$(c) \frac{1}{x} = \frac{2}{3}$$

$$\frac{1}{x+y} = \frac{2}{5}$$

$$\frac{1}{x+y+z} = 1$$

$$(d) \frac{1}{x} = 1\frac{1}{4}$$

$$\frac{1}{x} + \frac{1}{y} = 2\frac{1}{3}$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 3\frac{1}{2}$$

$$(e) \frac{3}{x} + \frac{1}{y} + \frac{4}{z} = 7$$

$$\frac{1}{x} + \frac{2}{y} + \frac{1}{z} = \frac{1}{2}$$

$$\frac{4}{x} - \frac{5}{y} - \frac{2}{z} = 12$$

(Hint: Let  $u = \frac{1}{x}$ ,  $v = \frac{1}{y}$ ,

$w = \frac{1}{z}$ )

$$(f) 2x + y + z + 7 = 0$$

$$x + 2y + z + 8 = 0$$

$$x + y + 2z + 9 = 0$$

$$(g) 6x - 3y + 2z = 5a - 2b$$

$$4x + 2y - z = 3(2a + 3b)$$

$$5x - 7y + 3z = -4(a + 3b)$$

$$(h) \frac{(x-z)}{4} - \frac{(x-y)}{3} = 0$$

$$\frac{(x-2)}{5} + \frac{(y+3)}{7} = \frac{x+y}{6}$$

$$\left(\frac{x}{2} - \frac{y}{3}\right) = 1 + \frac{y}{6} - \frac{z}{5}$$

### 3. One linear, one quadratic

Solve

$$(a) x^2 + xy = -1$$

$$x - y + 3 = 0$$

$$(b) y = x^2 - 4x - 2$$

$$2x + y = 1$$

$$(c) x + 3y = 9$$

$$xy = 6$$

$$(d) 3x - y = 2$$

$$y^2 - x^2 = 12$$

$$(e) 4xy + y^2 + 35 = 0$$

$$2x + 3y = 5$$

$$(f) x^2 + 2xy = 5$$

$$2x + 3y = 4$$

$$(g) x^2 + y^2 + 6x - 2y = 6$$

$$3x - 2y = 1$$

$$(h) 2ax - by = a^2$$

$$bxy = a^3$$

$$(i) x + 3y = 5$$

$$x^2 + y^2 = 25$$

#### 4. Simultaneous Log Equations

Solve

$$(a) \begin{cases} 3 \log_4 y + 2 \log_5 x = 3 \\ \log_5 x + \log_4 y = 1 \end{cases}$$

$$(c) \begin{cases} 2 \log_3 x + \log_3 y = 5 \\ \log_3 x - 2 \log_3 y = 5 \end{cases}$$

$$(b) \begin{cases} \log_3 x - \log_3 y = 4 \\ \log_2 (x - 33y) = 4 \end{cases}$$

$$(d) \begin{cases} 2 \log_3 x + \log_2 y = 10 \\ 4 \log_3 x - 3 \log_2 y = 0 \end{cases}$$

### 3. INEQUALITIES ( $>$ , $<$ , $\geq$ , $\leq$ )

#### 3.1 MANIPULATING INEQUALITIES

It is so easy to make a mistake when manipulating inequalities as certain operations reverse the inequality sign.

##### DO'S AND DON'TS

**Example:**  $8 > 4$

1. **Add 7:**  $8 + 7 > 4 + 7$  (Correct)
2. **Subtract 7:**  $8 - 7 > 4 - 7$  (Correct)
3. **Move:**  $8 - 4 > 0$  (Correct) and  $0 > 4 - 8$  (Correct)
4. **Multiply by 7:**  $56 > 28$  (Correct)
5. **Divide by 4:**  $2 > 1$  (Correct)
6. **Multiply by -7:**  $-56 > -28$  (Wrong)
7. **Divide by -4:**  $-2 > -1$  (Wrong)

##### TRICK

You can do all the same operations for inequalities as for equalities except two.

##### TRICK

Never multiply or divide an inequality by a negative number. Otherwise, do what you do for equalities.

#### 3.2 SOLVING INEQUALITIES

There are 4 types

[A] Linear

[B] Quadratic

[C] Rational

[D] Modulus

These are all done by the **region test**.

[A] **Linear:**  $ax + b > 0$

**STEPS**

1. Get letters on the side in order to give a positive coefficient and numbers on the other side.
2. Solve for the letter.

**Example 1:** Solve  $2x + 7 > 5x - 2$

SOLUTION

$$2x + 7 > 5x - 2 \Rightarrow -3x > -9 \Rightarrow 9 > 3x \Rightarrow 3 > x$$

[B] **Quadratics:**  $ax^2 + bx + c \leq 0$

**STEPS**

1. Get all terms on one side and zero on the other side.
2. Solve the corresponding equation to get the roots  $\alpha$ ,  $\beta$ .
3. Use the roots in ascending order to form regions:  $\leftarrow \alpha \leftrightarrow \beta \rightarrow$
4. Choose a nice number in each region to test the inequality using the test box.

**Example 2:** Solve  $2x^2 - 7x \leq 3$

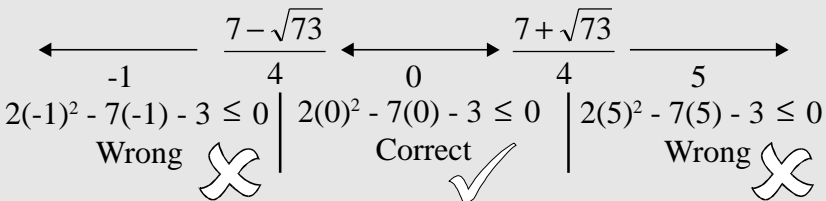
SOLUTION

$$2x^2 - 7x \leq 3$$

$$2x^2 - 7x - 3 \leq 0 \quad \text{.....Test Box}$$

$$\text{Solve } 2x^2 - 7x - 3 = 0 \text{ by magic } \Rightarrow x = \frac{7 \pm \sqrt{73}}{4}$$

Region Test on  $2x^2 - 7x - 3 \leq 0$



$$\therefore \frac{7 - \sqrt{73}}{4} \leq x \leq \frac{7 + \sqrt{73}}{4}$$

