

LC QUESTION

Find the area of the bounded region enclosed by the line $y = 2x - 1$, the line $x = 4$ and the curve $y = \frac{1}{x}$, where $x > 0$.

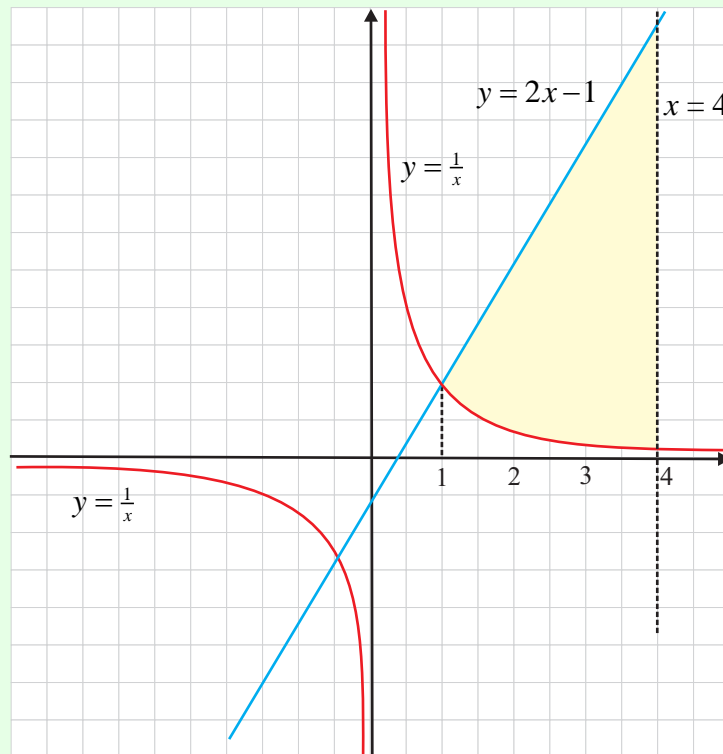
SOLUTION: Draw each curve.

$x = 4$ is a vertical line passing through $x = 4$ on the X-axis.

$y = 2x - 1$: $(0, -1), (\frac{1}{2}, 0)$ intercepts

$y = \frac{1}{x}$ is a rational curve. Asymptotes are $x = 0$ (Y-axis) and $y = 0$ (X-axis).

You need to find the yellow area bounded by the three curves.



You need to find out where $y = \frac{1}{x}$ and $y = 2x - 1$ intersect. Equate the two equations.

$$\therefore \frac{1}{x} = 2x - 1 \Rightarrow 2x^2 - x - 1 = 0 \Rightarrow (2x + 1)(x - 1) = 0$$

$$\Rightarrow x = -\frac{1}{2}, 1 \Rightarrow x = 1 (x > 0)$$

$$\text{Shaded area: } A = \int_1^4 (2x - 1) dx - \int_1^4 \frac{1}{x} dx = [x^2 - x - \ln x]_1^4$$

$$= [(4^2 - 4 - \ln 4) - (1^2 - 1 - \ln 1)] = 12 - \ln 4$$