Jason's Questions and Solutions

Question 1: Solve $x^2 - 4 \leq 0$.

Solution

Steps
1. Get all terms on one side and zero on the other side.
2. Solve the corresponding equation to get the roots α, β.
3. Carry out the region test. Use the roots in ascending order to form regions: $\leftarrow \alpha \leftrightarrow \beta \rightarrow$ Choose a nice number in each region to test the inequality using the **test box**.
4. Based on the region test write down the solutions.

1. $x^2 - 4 \leq 0$

2. Solve $x^2 - 4 = 0 \Rightarrow (x+2)(x-2) = 0 \Rightarrow x = -2,\ 2$

Roots: $\alpha = -2,\ \beta = 2$

3. Region Test on $\boxed{(x+2)(x-2) \leq 0}$ .....**Test Box**

←	-2	↔	2	→
-3		0		3
$(-1)(-5) \leq 0$		$(2)(-2) \leq 0$		$(5)(1) \leq 0$
Wrong		Correct		Wrong

4. $\therefore -2 \leq x \leq 2$

Question 2: If $(x-a)^2$ is a factor of $x^3 + 3px + q$ show that $p = -a^2$.

Solution

You can prove this result by using the division process.

$$
\begin{array}{r|l}
 & x + 2a \\
\hline
x^2 - 2ax + a^2 & x^3 + 0x^2 + 3px + q \\
 & \mp x^3 \pm 2ax^2 \mp a^2 x \\
\hline
 & \qquad 2ax^2 + (3p - q^2)x + q \\
 & \qquad \mp 2ax^2 \pm 4a^2 x \mp 2a^3 \\
\hline
 & \qquad\qquad (3p + 3a^2)x + (q - 2a^3)
\end{array}
$$

As $(x-a)^2$ is a factor, the remainder is zero, i.e. $0x + 0$.

$\Rightarrow 3p + 3a^2 = 0 \Rightarrow p + a^2 = 0 \Rightarrow p = -a^2$