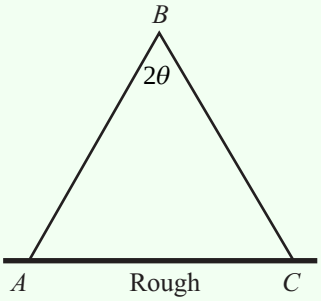
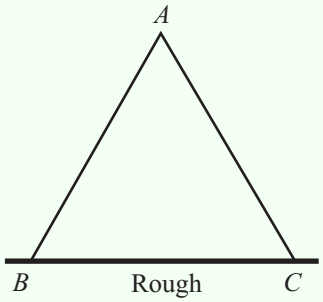
Exercise 6. Connected Bodies

1. Two identical rods *AB* and *BC* each of weight *W* are smoothly jointed at *B*. They rest in a vertical plane with *A* and *C* on a rough horizontal surface. The angle $\angle ABC = 2\theta$. The coefficient of friction at both *A* and *C* is μ. Show that $2\mu \geq \tan\theta$ and find the reaction at *B*.

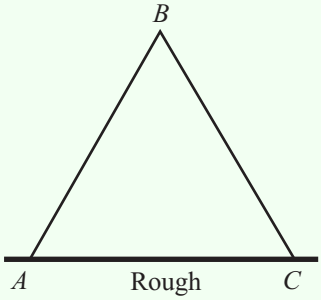


2. A step ladder consists of two equal arms *BA* with *AC* freely hinged at *A*. The weight of *AB* is three times that of *AC*. Show that if the angle between *BA* and *AC* is steadily increased then slipping first takes place at *C*. If *BA* and *AC* are to be placed so that $\angle BAC = 90^\circ$ find the least value of μ to prevent slipping.

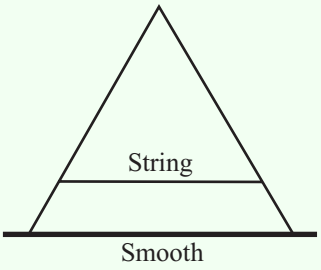


3. Two uniform rods *AB* and *BC* of equal length and of masses 5 kg and 3 kg respectively are freely hinged at *B*. *AB* and *BC* are in a vertical plane and the ends *A* and *C* are on rough horizontal ground. The coefficient of friction between each rod and the plane is the same. Find the normal reaction at *A* and *C*.

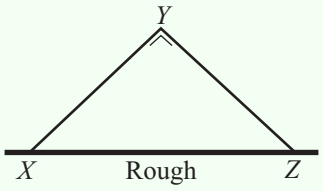
The angle $\angle ABC$ is increased until one of the rods begins to slip. Show that slipping will first occur at *C*.
Find the least value of the coefficient of friction if slipping has not occurred before $|\angle ABC| = 90^\circ$.



4. A step ladder consists of two identical legs hinged at the top, the centre of gravity of each leg being at its midpoint. It stands on a smooth level ground with a taut cord joining the points one quarter of the way up each leg. Show that if a man whose weight is twice the weight of the step-ladder stands three-quarters of the way up one side, the tension of the cord is quadrupled.

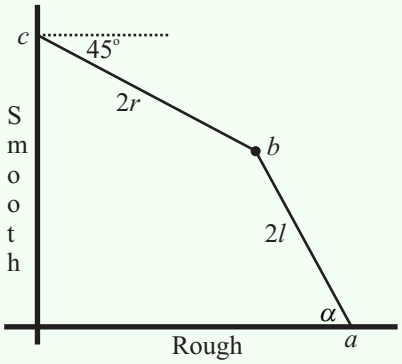


5. Two rods *XY* and *YZ* of equal length but of weights $\frac{1}{2}W$ and *W* respectively are freely hinged at *Y*. They stand in equilibrium in a vertical plane with the ends *X* and *Z* on a rough horizontal plane with $|\angle XYZ| = 90^\circ$. Show that the vertical components of the normal reaction of the plane at *X* and *Z* are $\frac{5}{8}W$ and $\frac{7}{8}W$ respectively.

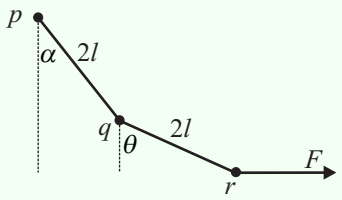


Show in separate diagrams the forces acting on each rod using vertical and horizontal components for the reaction at *Y*. Find all these forces and prove that if one rod is on the point of slipping the coefficient of friction is $\frac{3}{5}$.

6. Two uniform rods *ab* and *bc* of lengths $2l$ and $2r$ and of weights $2W$ and $3W$ respectively are smoothly hinged at *b* as shown. Find the coefficient of friction at *a* and show that $\tan\alpha = \frac{8}{3}$.



7. Two uniform rods *pq* and *qr* each of length $2l$ and weight *W* are freely joined together at *q* and hang freely from a fixed point *p*. A force *F* acting in a horizontal direction is applied to the rod *qr* and equilibrium is reached when *qr* makes an angle of $\tan^{-1} 2$ with the vertical.



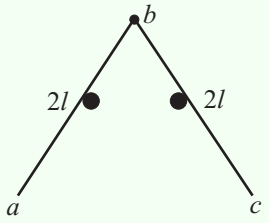
(i) Find the horizontal and vertical components of the action in the hinge *q*.

(ii) Show $F = W$.

(iii) Prove that the magnitude of the reaction at *p* is $\sqrt{5}W$.

(iv) If *pq* makes an angle α with the vertical show that $\tan\alpha = \frac{2}{3}$.

8. Two equal uniform rods *ab* and *bc* each of length $2l$ and weight *W* are freely jointed at *b* and rest in equilibrium in a vertical plane across two smooth horizontal pegs at the same horizontal level and $\frac{16}{27}l$ apart.

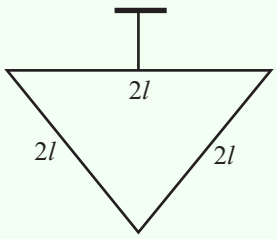


(i) Show in separate diagrams the forces on each rod.

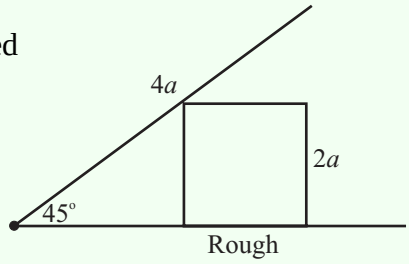
(ii) Show the inclination of each rod to the vertical is $\sin^{-1}(\frac{2}{3})$.

(iii) Determine the magnitude and direction of the reaction at *b*.

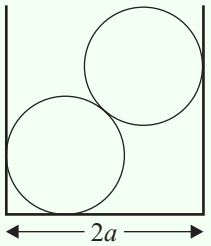
9. Three identical rods each of weight *W* are freely jointed together to form a triangle. The framework is suspended from the midpoint of one of the rods. Find the tension in the string with which the framework is suspended and the reactions at the joints.



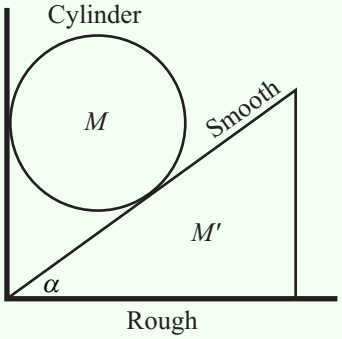
10. A uniform thin smooth rod of weight *W* and length $4a$ is freely hinged to a rough horizontal surface. The rod is lifted and a rough cube of side $2a$ and weight *W* is pushed under it until the rod is at 45° to the horizontal. Show that when released the system will remain at rest provided $7\mu \geq (2\sqrt{2} - 1)$ where μ is the coefficient of friction between the cube and the surface. Find the horizontal and vertical components of the reaction at the hinge in this position.



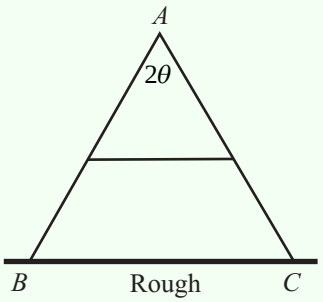
11. Two uniform identical spheres each of weight W and radius b rest inside a hollow smooth cylinder of radius a as shown with its base horizontal. If the spheres are smooth show that the reaction between each sphere and the curved surface is given by $\frac{(a-b)W}{\sqrt{2ab-a^2}}$. Show that the reaction between the spheres is $\frac{bW}{\sqrt{2ab-a^2}}$.



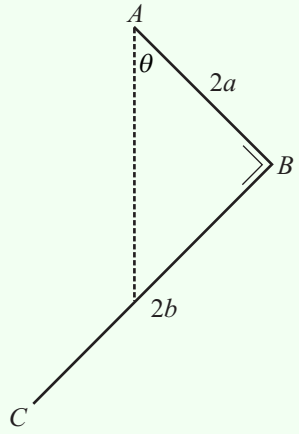
12. A uniform cylinder of mass M is supported against a smooth vertical wall by a smooth surface of a wedge of mass M' and angle α. The wedge can slide on a rough horizontal floor with coefficient of friction μ. If the wedge is on the point of slipping show that $\mu = \frac{M\tan\alpha}{M+M'}$.



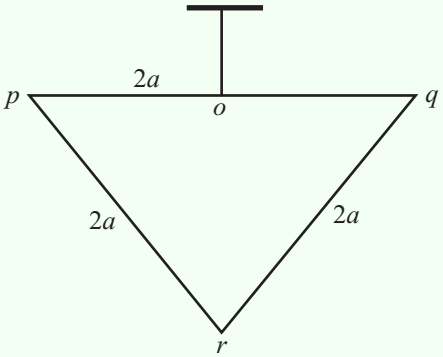
13. Two uniform rods AB and AC of equal length but weights W_1 and W_2 are smoothly hinged together at A and have their midpoints connected by a string. They stand in a vertical plane with B and C on a smooth horizontal plane. If $|\angle BAC| = 2\theta$ find the tension in the string.



14. An L-shaped structure is formed by joining two uniform rods AB and BC of length $2a$ and $2b$ and weights W and $5W$ respectively rigidly at B so $|\angle ABC| = 90^\circ$. The structure is supported from A. Show that $\tan\theta = \frac{10b}{11a}$.



15. An equilateral triangle pqr is formed from three uniform rods each of length $2a$ and weight W freely jointed at their ends. The triangle is freely suspended by a string attached to the midpoint o of pq so that it hangs symmetrically with r vertically below o. Find the horizontal and vertical components of the reactions at r.



EXERCISE 6
ANSWERS

1. $\frac{1}{2}W\tan\theta$ horizontally

2. $\frac{2}{3}$

3. $\frac{9}{2}g,\ \frac{7}{2}g,\ \mu=\frac{4}{7}$

5. $\frac{3}{8}W,\ \frac{1}{8}W$

6. $\frac{3}{10}$

7. **(i)** F, W

8. **(iii)** $\frac{1}{2}\sqrt{5}W$ horizontally

9. $3W,\ \frac{1}{6}\sqrt{3}W,\ \frac{1}{6}\sqrt{39}W$ at $\tan^{-1}2\sqrt{3}$ to the horizontal

10. $\frac{1}{4}\sqrt{2}W,\ (1-\frac{1}{4}\sqrt{2})W$

13. $\frac{1}{2}(W_1+W_2)\tan\theta$

15. $\dfrac{W}{2\sqrt{3}}$