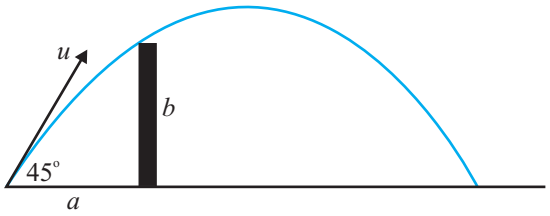
Leaving Cert Questions

2008

3. (a) A ball is projected from a point on the ground at a distance of a from the foot of a vertical wall of height b, the velocity of projection being u at an angle 45° to the horizontal.



If the ball just clears the wall prove that the greatest height reached is

$$\frac{a^2}{4(a-b)}.$$

(b) A particle is projected down an inclined plane with initial velocity u m/s.

The line of projection makes an angle of $2\theta^\circ$ with the inclined plane and the plane is inclined at θ° to the horizontal.

The plane of projection is vertical and contains the line of greatest slope.

The range of the particle on the inclined plane is $\frac{ku^2}{g}\sin\theta$.

Find the value of k.

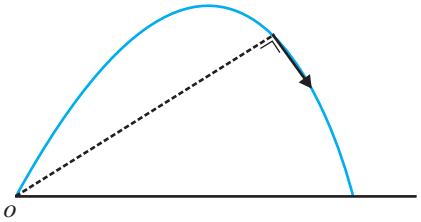
2007

3. (a) A particle is projected with a speed of $7\sqrt{5}$ m/s at an angle α to the horizontal.
Find the two values of α that will give a range of 12.5 m.

(b) A plane is inclined at an angle 45° to the horizontal. A particle is projected up the plane with initial speed u at an angle θ to the **horizontal**.
The plane of projection is vertical and contains the line of greatest slope.
The particle is moving horizontally when it strikes the inclined plane.
Show that $\tan\theta = 2$.

2006

3. (a) A particle is projected from a point o with velocity $9.8\vec{i} + 29.4\vec{j}$ m/s where $\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively.



(i) Express the velocity and displacement of the particle after t seconds in terms of $\vec{i}$ and $\vec{j}$.

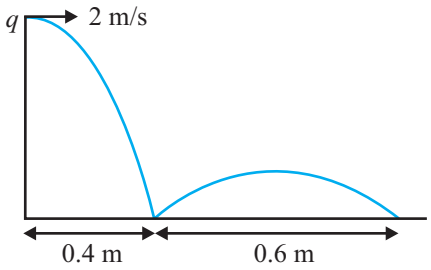
(ii) Find, in terms of t, the direction in which the particle is moving after t seconds.

(iii) Find the two times when the direction of the particle is at right angles to the line joining the particle to o.

(b) A particle is projected up an inclined plane with initial velocity u m/s.
The line of projection makes an angle 30^o with the plane and the plane is inclined at 30^o to the horizontal.
The plane of projection is vertical and contains the line of greatest slope.
Find, in terms of u, the range of the particle on the inclined plane.

2005

3. (a) A ball is projected horizontally from a point q above a smooth horizontal plane with speed 2 m/s.
The ball first hits the plane at a point whose horizontal displacement from q is 0.4 m.
The ball next strikes the plane at a horizontal displacement of 1 m from q.
The coefficient of restitution between the ball and the plane is e.
Find the value of e.



(b) A plane is inclined at an angle β to the horizontal. A particle is projected up the plane with initial velocity u at an angle α to the inclined plane.
The plane of projection is vertical and contains the line of greatest slope.

(i) Find the range of the particle on the inclined plane in terms of u, α and β.

(ii) Show that for a constant value of u the range is a maximum when

$$\alpha = 45^o - \tfrac{1}{2}\beta.$$

2004

3. (a) A particle is projected from a point on the horizontal floor of a tunnel with maximum height of 8 m. The particle is projected with an initial speed of 20 m/s inclined at an angle α to the horizontal floor.
Find, to the nearest metre, the greatest range which can be attained in the tunnel.

(b) A particle is projected up an inclined plane with initial velocity u m/s.
The line of projection makes an angle α **with the horizontal** and the inclined plane makes an angle θ with the horizontal. (The plane of projection is vertical and contains the line of greatest slope.)

If the particle strikes the inclined plane at right angles, show that

$$\tan\alpha = \frac{1+2\tan^2\theta}{\tan\theta}.$$

2003

3. (a) A particle is projected from a point on level horizontal ground at an angle θ to the horizontal ground.
Find θ, if the horizontal range of the particle is five times the maximum height reached by the particle.

(b) A particle is projected up an inclined plane with initial velocity u m/s.
The line of projection makes an angle α **with the horizontal** and the inclined plane makes an angle β with the horizontal. (The plane of projection is vertical and contains the line of greatest slope.)

Find, in terms of u, g, α and β, the range of the particle up the inclined plane.

2002

3. (a) A particle is projected from a point on the horizontal ground with a speed of 39.2 m/s inclined at an angle α to the horizontal ground. The particle is at a height of 14.7 m above the horizontal ground at times t_1 and t_2 seconds, respectively.

(i) Show that $t_2 - t_1 = \sqrt{64\sin^2\alpha - 12}$.

(ii) Find the value of α for which $t_2 - t_1 = \sqrt{20}$.

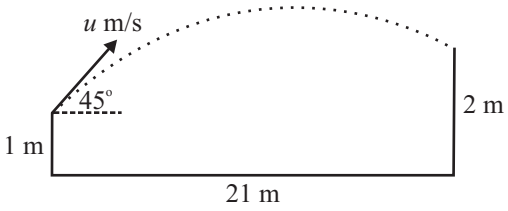
(b) A particle is projected with velocity u m/s at an angle θ **to the horizontal**, up a plane inclined at an angle β to the horizontal. (The plane of projection is vertical and contains the line of greatest slope). The particle strikes the plane at right angles.

(i) Show that $2\tan\beta\tan(\theta-\beta) = 1$.

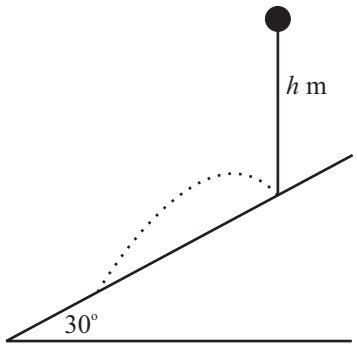
(ii) Hence, or otherwise, show that if $\theta = 2\beta$, the range of the particle up the inclined plane is $\frac{u^2}{g\sqrt{3}}$.

2001

3. (a) A player hits a ball with an initial speed of u m/s from a height of 1 m at an angle of 45° to the horizontal ground.
A member of the opposing team, 21 m away, catches the ball at a height of 2 m above the ground.
Find the value of u.



(b) A ball is dropped from a height of h m onto a smooth inclined plane. The ball strikes the plane at p and rebounds.
The plane is inclined at an angle of 30° to the horizontal and the coefficient of restitution between the ball and the plane is $\frac{1}{2}$.



Find how far down the plane from p is the ball's next point of impact.
Express your answer in terms of h.

2000

3. (a) A particle is projected with a velocity of u m/s at an angle β to the horizontal ground.

Show that the particle hits the ground at a distance $\dfrac{u^2 \sin 2\beta}{g}$ from the point of projection.
Find the angle of projection which gives maximum range.

(b) A particle is projected at an angle $\alpha = \tan^{-1} 3$ **to the horizontal** up a plane inclined at an angle θ to the horizontal. (The plane of projection is vertical and contains the line of greatest slope). The particle strikes the plane at right angles.
Find two possible values for θ.

1999

A particle is projected from a point p up an inclined plane with a speed of $4g\sqrt{2}$ m/s at an angle $\tan^{-1}(\frac{1}{3})$ to the inclined plane. The plane is inclined at an angle θ to the horizontal. (The plane of projection is vertical and contains the line of greatest slope). The particle is moving horizontally when it strikes at the point q.

(a) Find the two possible values for θ.

(b) If $\tan\theta = 0.5$ then
(i) find the magnitude of the velocity with which the particle strikes the inclined plane at q.

(ii) determine the total energy at p and show that it is equal to the total energy at q.

1998

3. (a) A football is kicked from a spot on level ground with a velocity of $\sqrt{8g}$ m/s and strikes a vertical wall 4 m away at a point 2 m above the ground. Find the two possible angles of projection.

(b) A particle is projected down a slope which is inclined at 45^o to the horizontal. The particle is projected from a point on the slope and has an initial velocity of $7\sqrt{2}$ m/s at an angle α to the inclined plane. Find the value of α if

(i) the particle first hits the slope after 2 seconds

(ii) the landing angle when the slope $\tan^{-1}(\frac{1}{3})$.

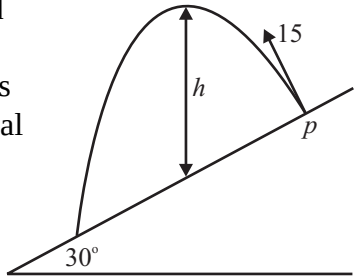
1997

3. (a) A golf ball, at rest on horizontal ground, is struck so that it starts to move with velocity $3u\vec{i} + u\vec{j}$ where $\vec{i}$ and $\vec{j}$ are unit vectors along and perpendicular to the ground, respectively. In its flight the ball rises to a maximum height of 15 m.
Calculate

(i) the value of u,

(ii) the magnitude and direction of the velocity with which the ball strikes the ground.

(b) A particle is projected from a point p with initial speed 15 m/s, down a plane inclined at an angle of 30^o to the horizontal. The direction of projection is at right angles to the inclined plane. (The plane of projection is vertical and contains the line of greatest slope). Find



(i) the perpendicular height of the particle above the plane after t seconds and hence, or otherwise, show that the vertical height h of the particle above the plane after t seconds is

$$10\sqrt{3}t - 4.9t^2$$

(ii) the greatest vertical height it attains above the plane (i.e. the maximum value of h) correct to two places of decimal.

1996

3. (a) A particle is projected from the ground with a velocity of 50.96 m/s at an angle $\tan^{-1}(\frac{5}{12})$ to the horizontal. On its upward path it just passes over a wall 14.7 m high. During its flight it also passes over a second wall 18.375 m high.
Show that the second wall must be not less than 23.52 m and not more than 70.56 m from the first wall.

(b) A plane is inclined at an angle of 2β to the vertical. A particle is projected up the plane with initial velocity $u\cos\beta$ at an angle β to the inclined plane. The plane of projection is vertical and contains the line of greatest slope.
Show

(i) that the time of flight of the particle is $\frac{u}{g}$

(ii) that the range of the particle on the plane is $\frac{u^2}{2g}$.

Answers
Leaving Cert. Questions

2008. 3 (b) $k = 4$

2007. 3 (a) 15^o, 75^o

2006. 3 (a) (i) $\vec{r} = 9.8t\,\vec{i} + (29.4t - \frac{1}{2}gt^2)\,\vec{j}$, $\vec{v} = 9.8\vec{i} + (29.4 - gt)\,\vec{j}$

(ii) $\tan^{-1}(3-t)$

(iii) 4 s, 5 s

(b) $\dfrac{2u^2}{3g}$

2005. 3 (a) $e = \frac{3}{4}$ (b) (i) $\dfrac{2u^2 \sin\alpha \cos(\alpha+\beta)}{g\cos^2\beta}$

2004. 3 (a) 40 m

2003. 3 (a) $\tan^{-1}(\frac{4}{5}) = 38.7^o$ (b) $\dfrac{u^2}{g\cos^2\beta}\{\sin(2\alpha-\beta) - \sin\beta\}$

2002. 3 (a) (ii) 45^o

2001. 3 (a) 14.7 m s^{-1} (b) $\frac{3}{2}h$

2000. 3 (a) 45^o (b) $\tan^{-1}(\frac{1}{2})$, 45^o

1999. 3 (a) $\tan^{-1}(\frac{1}{2})$, 45^o (b) (i) $4g$, (ii) $16mg^2 = 8mg^2 + 8mg^2$

1998. 3 (a) $\tan^{-1}3$, 45^o (b) (i) 45^o, (ii) 44.4^o

1997. 3 (a) (i) $\sqrt{30g}$, (ii) $\sqrt{300g}$ at $\tan^{-1}(\frac{1}{3})$ (b) (ii) 15.31 m