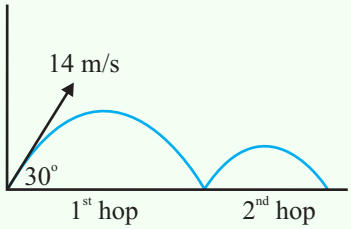
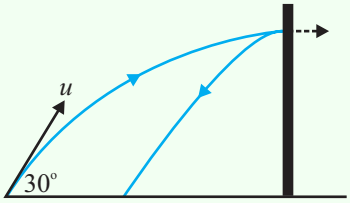
Exercise 8. Projectiles and Elasticity

1. A particle is projected from point *o* with speed 14 m s^{-1} at 30^{o} to the horizontal. Find
(i) the time of flight,
(ii) horizontal range,
(iii) horizontal and vertical components of velocity on landing.

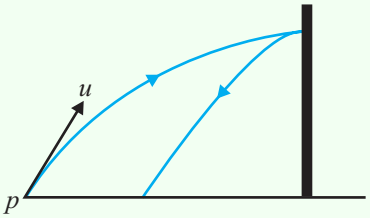
If $e = \frac{1}{2}$ is the coefficient of elasticity between the particle and floor, find the range of the second hop.



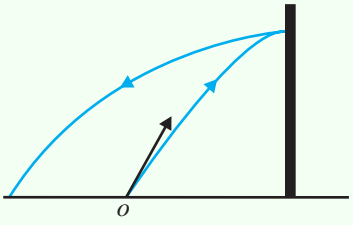
2. A ball thrown at an angle of 30^{o} to the horizontal strikes a wall at right angles after 0.5 s. If the coefficient of restitution is $\frac{2}{7}$ how far from its starting point does the ball strike the horizontal.



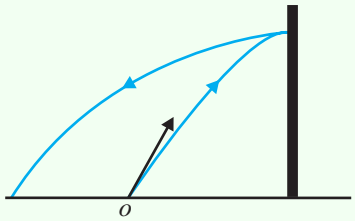
3. From a point *p* on horizontal ground an elastic particle is projected with velocity $\vec{u} = (28\vec{i} + 21\vec{j})$ m s^{-1} where $\vec{i}$ and $\vec{j}$ are unit vectors along and perpendicular to the plane respectively. Find the time it takes to get to the highest point and the displacement and velocity at this point. If at this point the particle strikes a fixed vertical wall where the coefficient of restitution is $\frac{1}{2}$ find how far from the point of projection the particle strikes the ground.



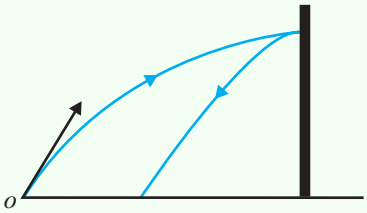
4. A particle is projected from a point *o* on horizontal ground with velocity $(14\vec{i} + 28\vec{j})$ m s^{-1}. Find its velocity and displacement after 2 seconds. If after this time the particle strikes a vertical wall, find the time to hit the ground and how far from the point of projection it lands if the coefficient of elasticity $e = \frac{1}{2}$.



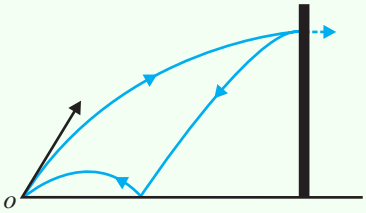
5. A ball is projected with velocity $(u\vec{i} + v\vec{j})$ m s^{-1} from a point *o* on horizontal ground. Find its velocity and displacement after time *t*. If after this time the ball strikes a vertical wall and rebounds, show that the time to reach the ground is $\dfrac{2v - gt}{g}$ and that it strikes the ground a distance $eu\left(\dfrac{2v - gt}{g}\right)$ where *e* is the coefficient of elasticity between the wall and the ball.



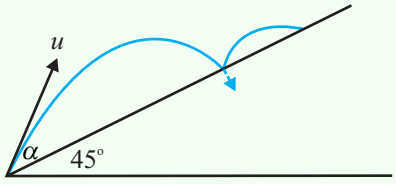
6. A ball is projected with velocity $(u\vec{i} + v\vec{j})$ m s^{-1} from a point *o* on horizontal ground. Find the time to reach the highest point. Find its velocity and displacement there. ..... If the ball strikes a vertical wall find the time to reach the ground and the distance from *o* it strikes the ground if the coefficient of elasticity between the ball and the wall is *e*.



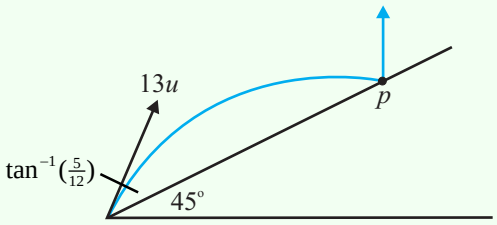
7. A particle is projected from a point o on a smooth horizontal ground so as to strike a vertical wall horizontally. If the particle returns to A after only one bounce on the ground show that $e + 2ee' = 1$ where *e* and e' are the coefficients of restitution at the wall and ground respectively.



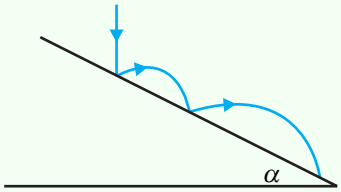
8. From the foot of a fixed plane inclined at 45° to the horizontal a particle is projected with speed *u* at an angle α to the plane. Find $\tan\alpha$ if the particle strikes the plane at right angles. If the coefficient of restitution between the particle and the plane is $\frac{1}{2}$ find the distance from the original point where it strikes the plane again.



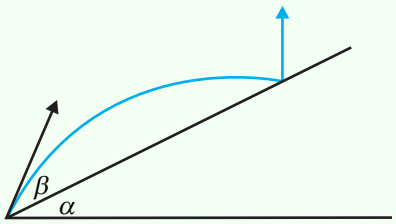
9. A particle is projected up an inclined plane with speed $13u$. The angle of projection is $\tan^{-1}(\frac{5}{12})$ with the plane. The plane is inclined at 45° to the horizontal. The particle strikes the plane at p. Find its velocity at p. If the coefficient of restitution between the particle and the plane is 0.4 show that the particle bounces up vertically.



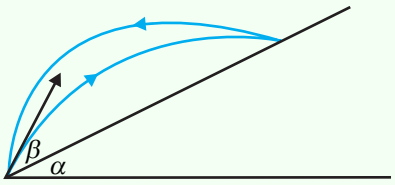
10. A perfectly elastic particle falls vertically on a smooth plane inclined at an angle α to the horizontal and rebounds hopping down the plane. Show that the length of the first hop is $\dfrac{4u^2 \sin\alpha}{g}$ and the length of the second hop is double this.



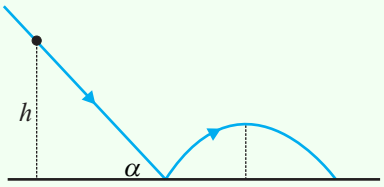
11. A particle is projected up a plane inclined at α to the horizontal. The angle of projection with the inclined plane is β. The particle hits the plane and rebounds vertically upwards. If e is the coefficient of restitution between the particle and the plane show that $\cot\beta = (e+2)\tan\alpha$.



12. A particle is projected at angle β to a smooth plane inclined at angle α to the horizontal. It returns to its initial position after one bounce on the plane. Prove $\cot\beta = (1+e)\tan\alpha$.



13. A particle slides from rest down a smooth plane of inclination α and at the bottom impinges on a smooth horizontal plane. If the original height of the particle above the horizontal plane is h, show that after the first bounce the greatest height attained is $he^2 \sin^2 \alpha$ where e is the coefficient of restitution and the horizontal plane. Find the range of the first bounce.



Answers

Exercise 8

1. (i) $\frac{10}{7}$ s (ii) $10\sqrt{3}$ m (iii) $7\sqrt{3}$ m s^{-1}, -7 m s^{-1}; $5\sqrt{3}$ m

2. 1.2 m

3. $\frac{15}{7}$ s, $(60\vec{i} + 22.5\vec{j})$ m, $28\vec{i}$ m s^{-1}, 30 m

4. $(14\vec{i} + 8.4\vec{j})$ m s^{-1}, $(28\vec{i} + 36.4\vec{j})$ m, $\frac{26}{7}$ s, 26 m to the left of o

5. $(u\vec{i} + (v - gt)\vec{j})$ m s^{-1}, $(ut\vec{i} + (vt - \frac{1}{2}gt^2)\vec{j})$ m

6. $\frac{v}{g}$ s, $u\vec{i}$ m s^{-1}, $(\frac{uv}{g}\vec{i} + \frac{v^2}{2g}\vec{j})$ m, $\frac{v}{g}$ s, $\frac{uv}{g}(1 - e)$ m

8. $\tan\alpha = \frac{1}{2}$, $\dfrac{3u^2}{5g\sqrt{2}}$ m

9.

13. $(2eh \sin 2\alpha)$ m