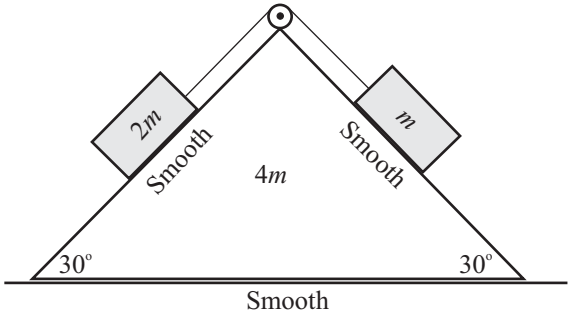
WORKED EXAMPLES

EXAMPLE 1 [LC 2008]

Particles of mass $2m$ and m are connected by a light inextensible string which passes over a smooth pulley at the vertex of a wedge-shaped block, one particle resting on each of the smooth faces.
The mass of the wedge is $4m$ and the inclination of each face to the horizontal is 30°. The wedge rests on a smooth horizontal surface and the system is released from rest.

(i) Show, on separate diagrams, the forces acting on the wedge and on the particles.

(ii) Find the acceleration of the wedge.



SOLUTION: Page 2

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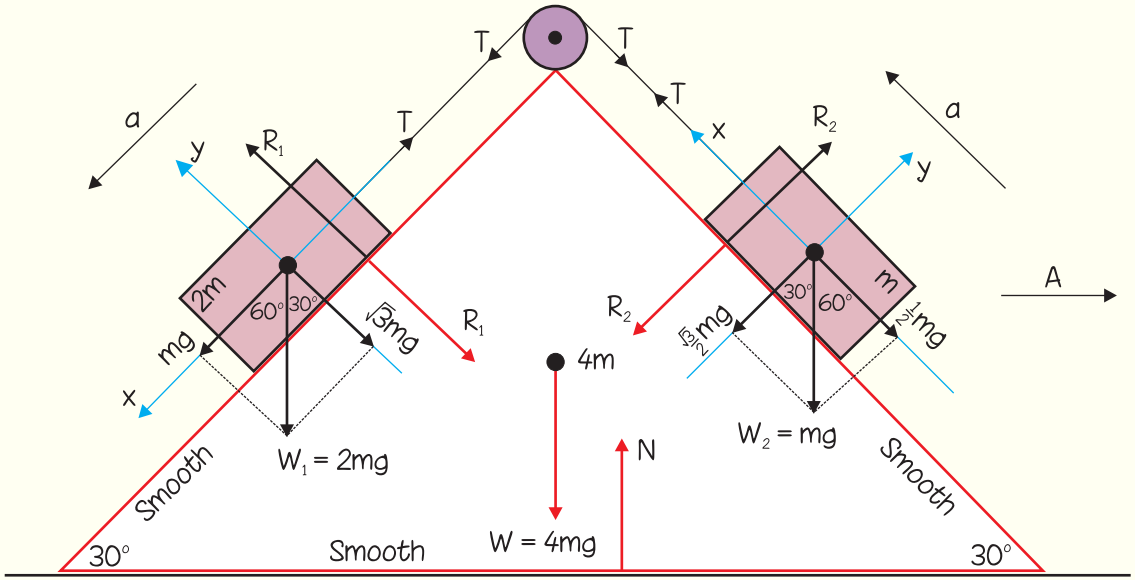
SOLUTION



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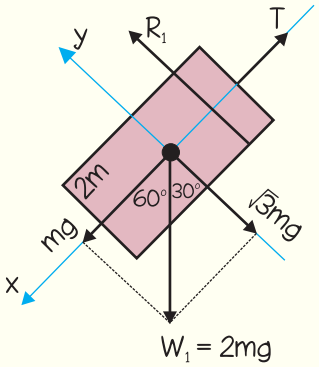
SOLUTION

Forces on the $2m$ mass

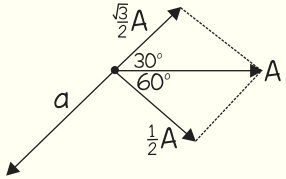


$W_1 = 2mg$

$W_{1x} = 2mg\cos 60^\circ = 2mg \times \frac{1}{2} = mg$

$W_{1y} = 2mg\cos 30^\circ = 2mg \times \frac{\sqrt{3}}{2} = \sqrt{3}mg$

Acceleration of the $2m$ mass



$A_x = A\cos 30^\circ = \frac{\sqrt{3}}{2}A$

$A_y = A\cos 60^\circ = \frac{1}{2}A$

MATHEMATICAL CALCULATIONS

$F_{\text{Net}} = ma$

x direction:

$mg - T = 2m(a - \frac{\sqrt{3}}{2}A)$

$mg - T = 2ma - \sqrt{3}mA$....**(1)**

y direction:

$R_1 - \sqrt{3}mg = 2m(-\frac{1}{2}A)$

$R_1 - \sqrt{3}mg = -mA$....**(2)**

CONT....

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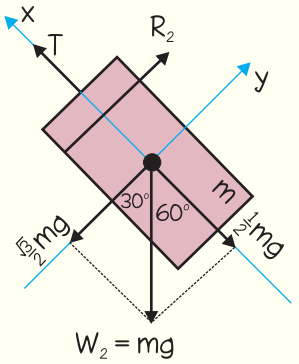
SOLUTION

Forces on the m mass

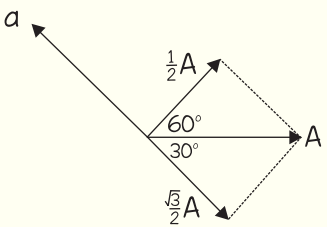


$W_1 = 2mg$

$W_{2x} = mg\cos 60^\circ = mg \times \frac{1}{2} = \frac{1}{2}mg$

$W_{1y} = mg\cos 30^\circ = mg \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}mg$

Acceleration of the m mass



$A_x = A\cos 30^\circ = \frac{\sqrt{3}}{2}A$

$A_y = A\cos 60^\circ = \frac{1}{2}A$

MATHEMATICAL CALCULATIONS

$F_{\text{Net}} = ma$

x direction:

$T - \frac{1}{2}mg = m(a - \frac{1}{2}A)$

$T - \frac{1}{2}mg = ma - \frac{1}{2}mA$...(**3**)

y direction:

$R_2 - \frac{\sqrt{3}}{2}mg = \frac{1}{2}mA$...(**4**)

CONT....

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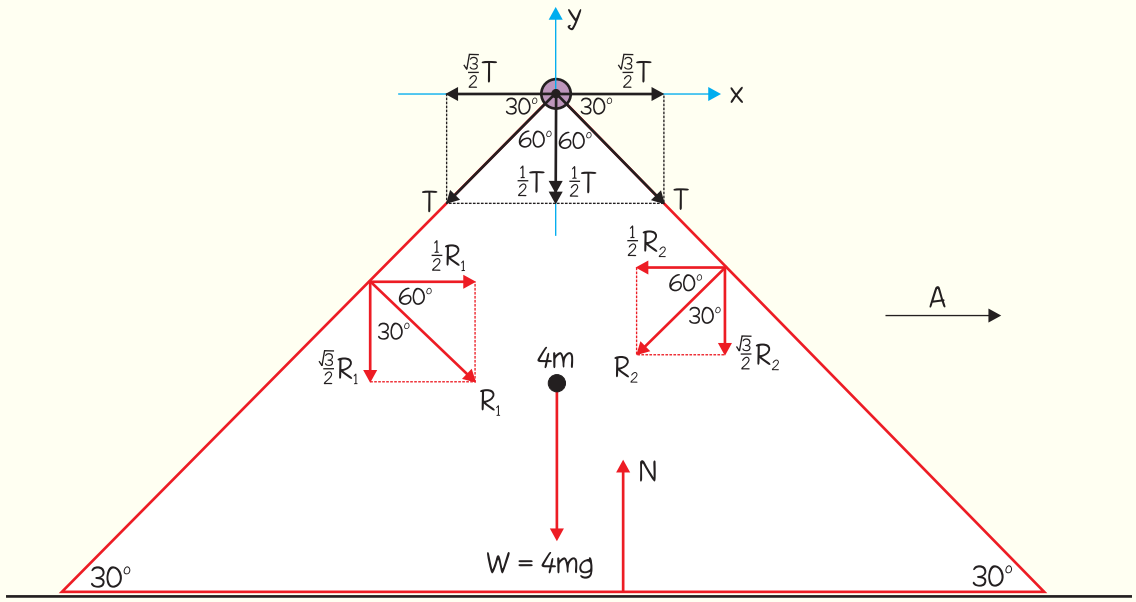
(ii) Find the acceleration of the wedge.

SOLUTION

Forces on the wedge



MATHEMATICAL CALCULATIONS

$F_{\text{Net}} = ma$

x direction:

$$\frac{\sqrt{3}}{2}T - \frac{\sqrt{3}}{2}T + \frac{1}{2}R_1 - \frac{1}{2}R_2 = 4mA$$

$$R_1 - R_2 = 8mA \ldots (5)$$

y direction:

$$N - 4mg - \frac{1}{2}T - \frac{1}{2}T = 0$$

$$N - 4mg - T = 0 \ldots (6)$$

CONT....

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(ii) Find the acceleration of the wedge.

SOLUTION

MATHEMATICAL CALCULATIONS

$mg - T = 2ma - \sqrt{3}mA$....**(1)**

$R_1 - \sqrt{3}mg = -mA$.....**(2)**

$T - \frac{1}{2}mg = ma - \frac{1}{2}mA$.....**(3)**

$R_2 - \frac{\sqrt{3}}{2}mg = \frac{1}{2}A$.....**(4)**

$R_1 - R_2 = 8mA$.....**(5)**

$N - 4mg - T = 0$.....**(6)**

$R_1 - \sqrt{3}mg = -mA$.....**(2)**

$R_1 = \sqrt{3}mg - mA$

$R_2 - \frac{\sqrt{3}}{2}mg = \frac{1}{2}A$.....**(4)**

$R_2 = \frac{\sqrt{3}}{2}mg + \frac{1}{2}A$

$R_1 - R_2 = 8mA$.....**(5)**

$\sqrt{3}mg - mA - \frac{\sqrt{3}}{2}mg - \frac{1}{2}A = 8mA$

$\frac{\sqrt{3}}{2}mg = \frac{19}{2}mA$

$\sqrt{3}g = 19A$

$\therefore A = \dfrac{\sqrt{3}g}{19}\ \text{m s}^{-2}$