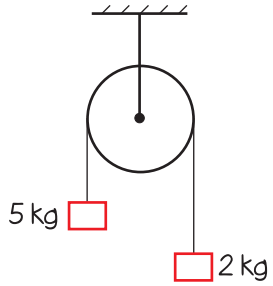
WORKED EXAMPLES

EXAMPLE 1

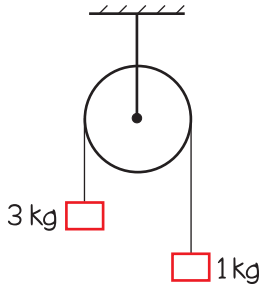
The pulley is smooth. Find the acceleration of each particle and the tension in the string.



SOLUTION: Page 2

EXAMPLE 2

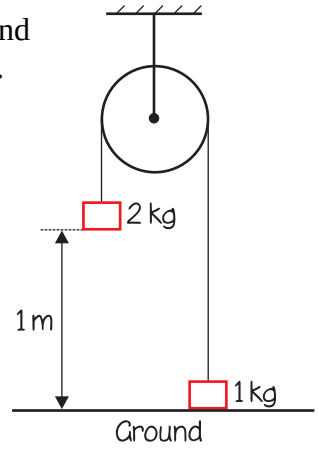
The pulley is smooth. Find the acceleration of each particle and the tension in the string. If the system is released from rest find the velocity of each (i) after 3 s, (ii) after travelling 1 m.



SOLUTION: Page 3

EXAMPLE 3

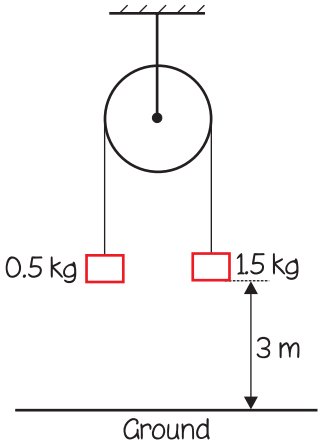
The pulley is smooth. The 1 kg mass is on the ground and the system is released from rest. Find the maximum height reached by the 1 kg mass.



SOLUTION: Page 4

EXAMPLE 4

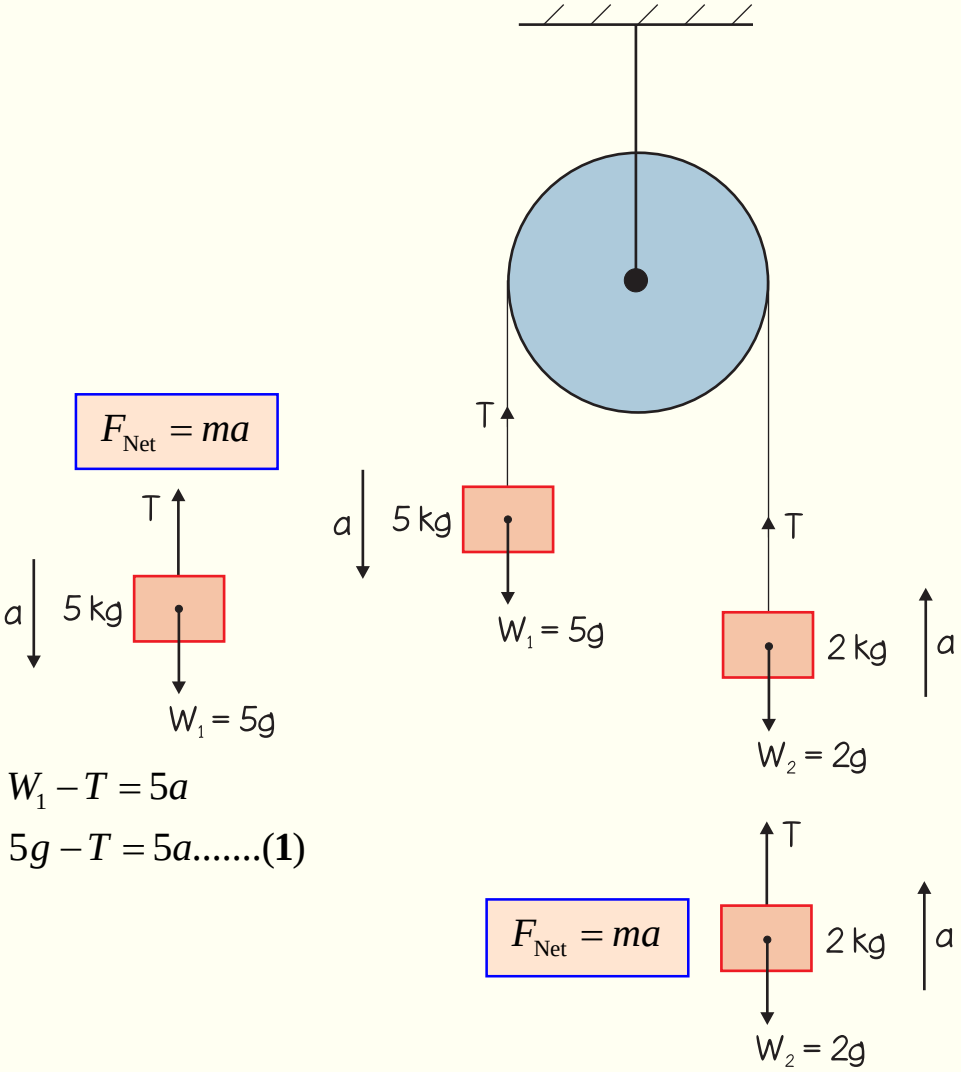
The pulley is smooth. The system is released from rest with the 1.5 kg mass 3 m above the ground. Find the acceleration of each particle as the 1.5 kg mass is moving towards the ground. Find the time for which the 1.5 kg particle is on the ground.



SOLUTION: Page 7

EXAMPLE 1: The pulley is smooth. Find the acceleration of each particle and the tension in the string.

SOLUTION



$F_{\text{Net}} = ma$

$W_1 - T = 5a$

$5g - T = 5a.....(\mathbf{1})$

$F_{\text{Net}} = ma$

$T - W_2 = 2a$

$T - 2g = 2a.....(\mathbf{2})$

MATHEMATICAL CALCULATIONS

$5g - T = 5a.....(\mathbf{1})$

$T - 2g = 2a.....(\mathbf{2})$

$3g \quad = 7a \Rightarrow a = \frac{3}{7}g \text{ ms}^{-2}$

$T - 2g = 2a...(\mathbf{2})$

$T = 2a + 2g$

$= 2(\frac{3}{7}g) + 2g$

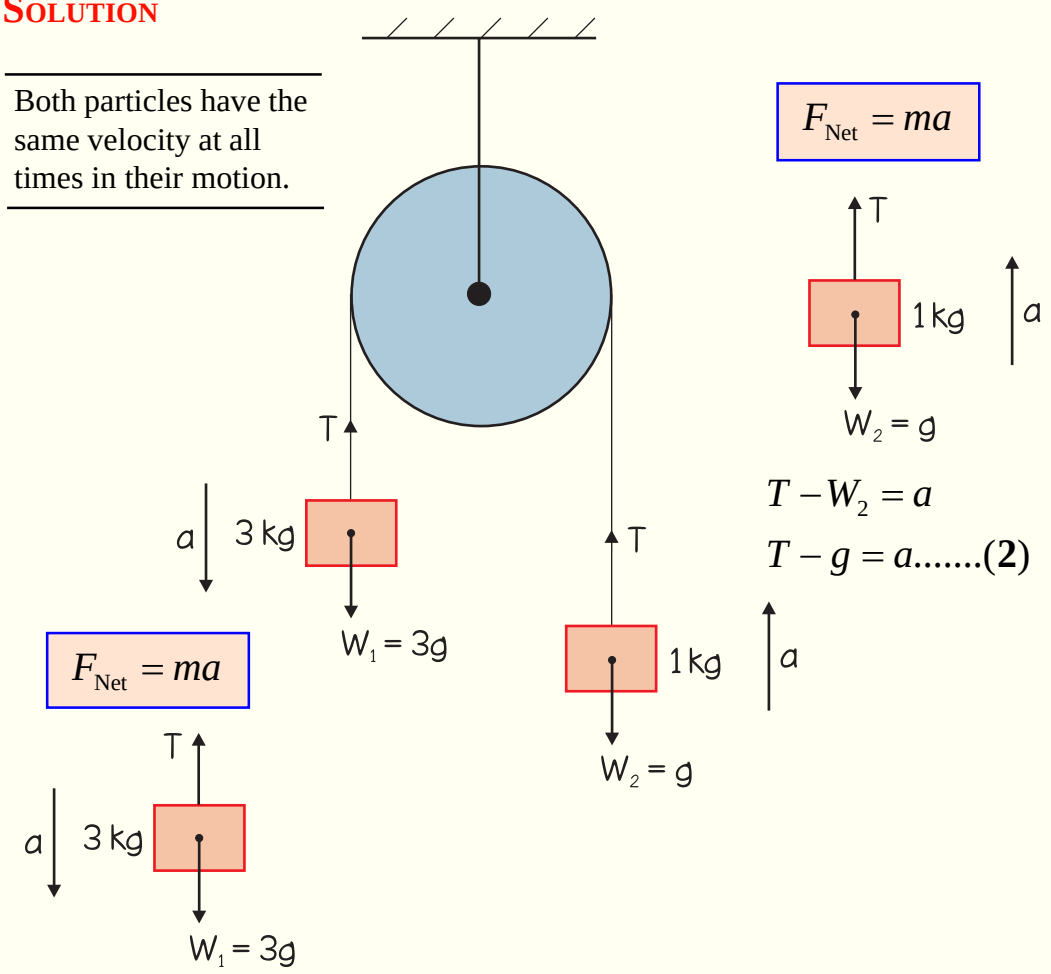
$= \frac{6}{7}g + 2g$

$= \frac{20}{7}g \text{ N}$

EXAMPLE 2: The pulley is smooth. Find the acceleration of each particle and the tension in the string. If the system is released from rest find the velocity of each (i) after 3 s, (ii) after travelling 1 m.

SOLUTION

Both particles have the same velocity at all times in their motion.



$F_{\text{Net}} = ma$

$W_1 - T = 3a$

$3g - T = 3a.....(\mathbf{1})$

$F_{\text{Net}} = ma$

$T - W_2 = a$

$T - g = a.....(\mathbf{2})$

MATHEMATICAL CALCULATIONS

$$3g - T = 3a.....(\mathbf{1})$$
$$T - g = a.....(\mathbf{2})$$
$$2g = 4a \Rightarrow a = \tfrac{1}{2}g \text{ ms}^{-2}$$

$T - g = a...(\mathbf{2})$

$T = a + g$

$= (\tfrac{1}{2}g) + g$

$= \tfrac{3}{2}g$ N

$u = 0$ ms^{-1}

$a = \tfrac{1}{2}g$ ms^{-2}

$t = 3$ s

$v = ?$

(i) $v = u + at$

$= 0 + (\tfrac{1}{2}g)(3) = \tfrac{3}{2}g$ ms^{-1}

$u = 0$ ms^{-1}

$a = \tfrac{1}{2}g$ ms^{-2}

$s = 1$ m

$v = ?$

(ii) $v^2 = u^2 + 2as$

$= 0^2 + 2(\tfrac{1}{2}g)(1)$

$= g$

$\therefore v = \sqrt{g}$ ms^{-1}

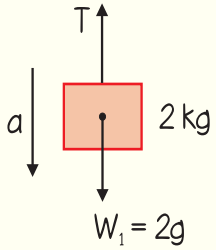
EXAMPLE 3: The pulley is smooth. The 1 kg mass is on the ground and the system is released from rest. Find the maximum height reached by the 1 kg mass.

SOLUTION

$F_{\text{Net}} = ma$

$W_1 - T = 2a$

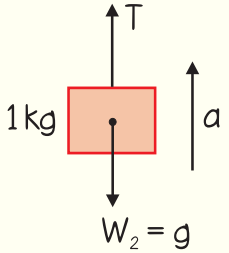
$2g - T = 2a$.....**(1)**

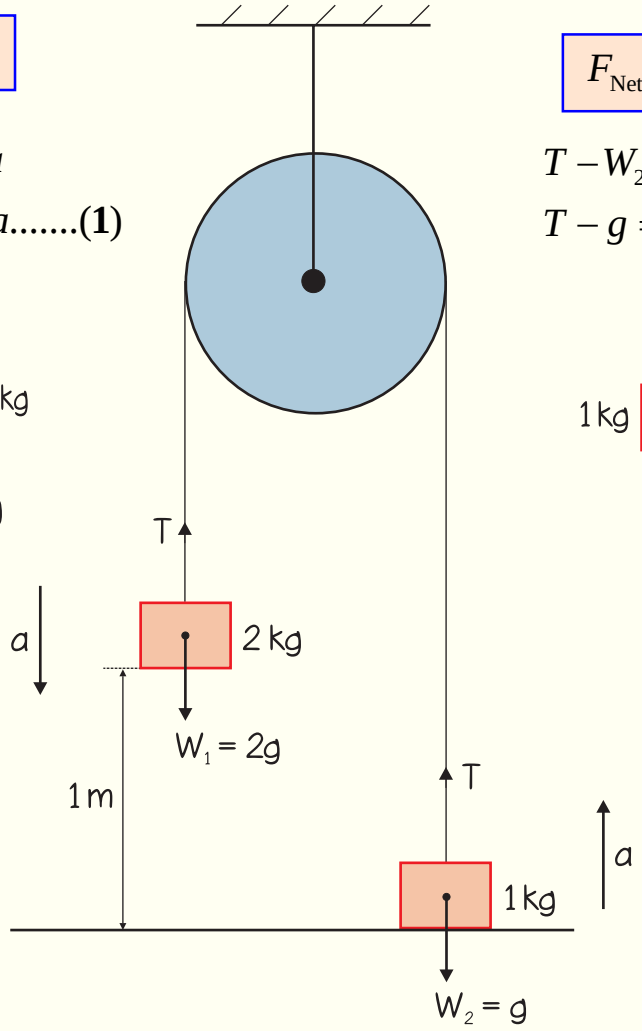


$F_{\text{Net}} = ma$

$T - W_2 = a$

$T - g = a$.....**(2)**





MATHEMATICAL CALCULATIONS

$2g - T = 2a$.....**(1)**

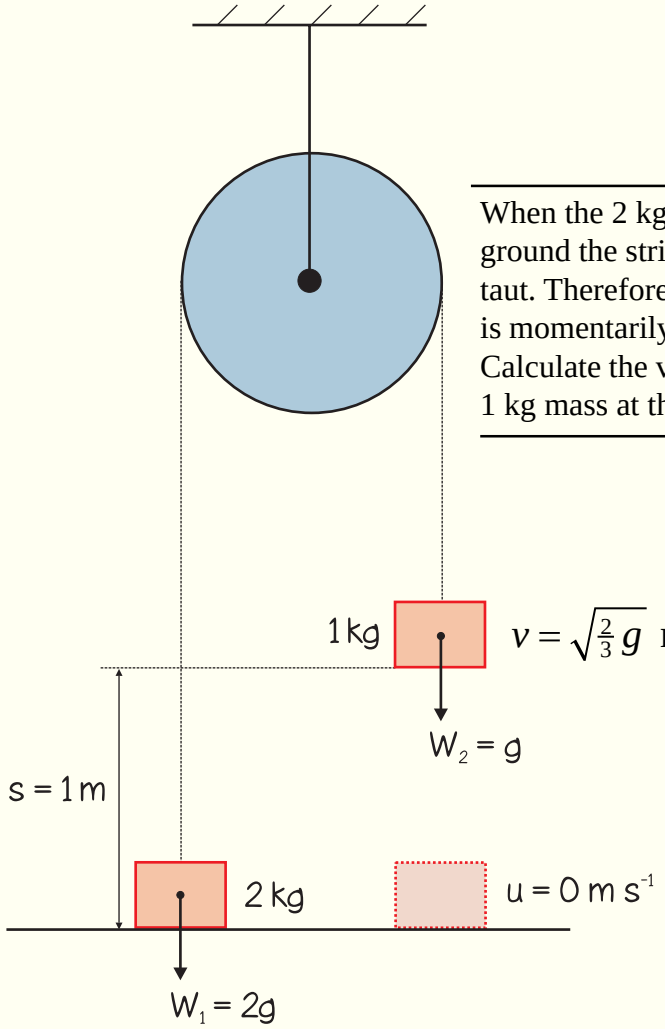
$T - g = a$.....**(2)**

$g = 3a \Rightarrow a = \frac{1}{3}g \text{ ms}^{-2}$

CONT.....

EXAMPLE 3: The pulley is smooth. The 1 kg mass is on the ground and the system is released from rest. Find the maximum height reached by the 1 kg mass.

SOLUTION



When the 2 kg mass hits the ground the string is no longer taut. Therefore, the 1 kg mass is momentarily in free fall. Calculate the velocity of the 1 kg mass at this instant.

MATHEMATICAL CALCULATIONS

$$2g - T = 2a.....(1)$$
$$T - g = a.....(2)$$
$$g = 3a \Rightarrow a = \tfrac{1}{3}g \text{ ms}^{-2}$$

1 kg particle:

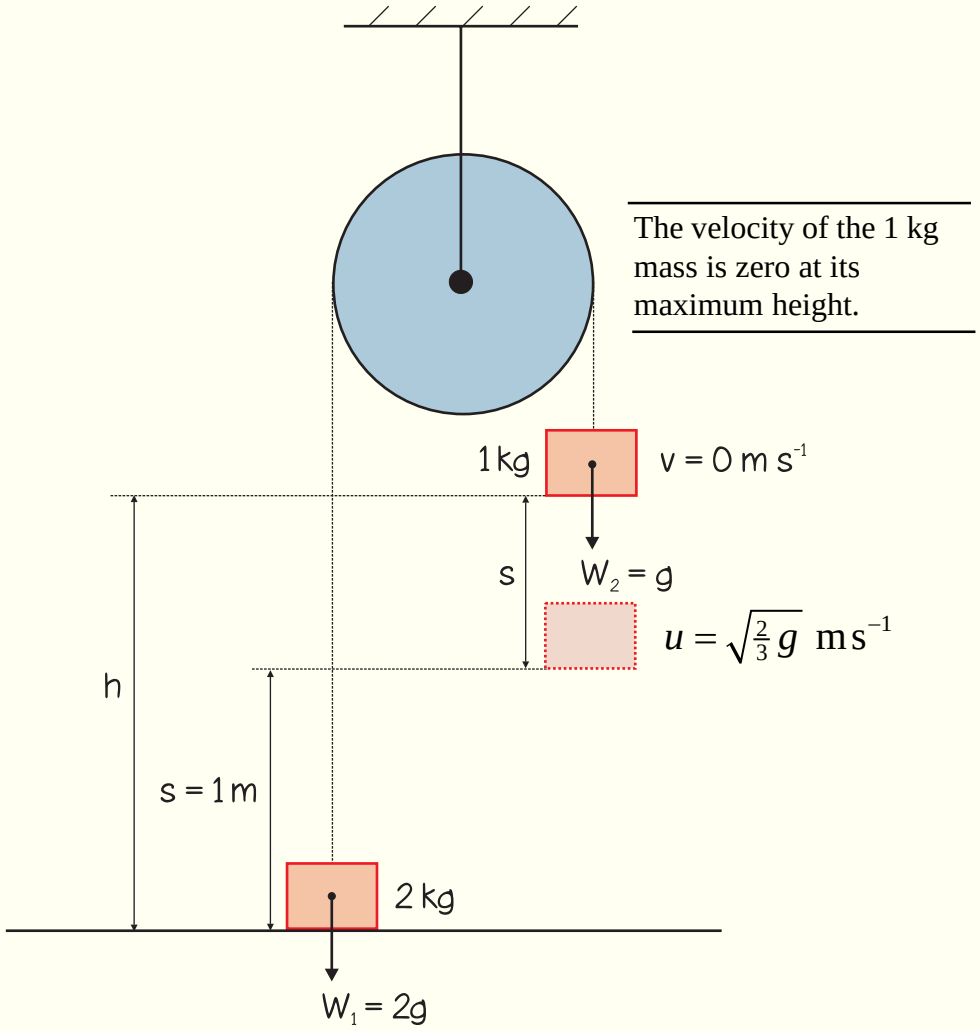
$u = 0$ ms^{-1}
$s = 1$ m
$a = \frac{1}{3}g$ ms^{-2}
$v = ?$

$$v^2 = u^2 + 2as$$
$$= 0^2 + 2(\tfrac{1}{3}g)(1)$$
$$= \tfrac{2}{3}g$$
$$\therefore v = \sqrt{\tfrac{2}{3}g} \text{ ms}^{-1}$$

CONT.....

EXAMPLE 3: The pulley is smooth. The 1 kg mass is on the ground and the system is released from rest. Find the maximum height reached by the 1 kg mass.

SOLUTION



The velocity of the 1 kg mass is zero at its maximum height.

MATHEMATICAL CALCULATIONS

$$2g - T = 2a \text{.....(1)}$$
$$T - g = a \text{.....(2)}$$
$$g = 3a \Rightarrow a = \tfrac{1}{3}g \text{ ms}^{-2}$$

1 kg particle:

$u = 0 \text{ ms}^{-1}$
$s = 1 \text{ m}$
$a = \tfrac{1}{3}g \text{ ms}^{-2}$
$v = ?$

$$v^2 = u^2 + 2as$$
$$= 0^2 + 2(\tfrac{1}{3}g)(1)$$
$$= \tfrac{2}{3}g$$
$$\therefore v = \sqrt{\tfrac{2}{3}g} \text{ ms}^{-1}$$

$u = \sqrt{\tfrac{2}{3}g} \text{ ms}^{-1}$
$v = 0 \text{ ms}^{-1}$
$a = -g \text{ ms}^{-2}$
$s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = \left(\sqrt{\tfrac{2}{3}g}\right)^2 + 2(-g)s$$
$$0 = \tfrac{2}{3}g - 2gs$$
$$2gs = \tfrac{2}{3}g$$
$$s = \tfrac{1}{3} \text{ m}$$

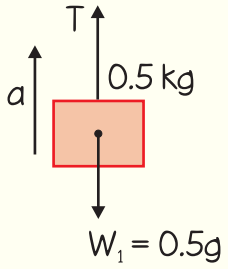
Maximum height h:

$$h = 1 + \tfrac{1}{3} = \tfrac{4}{3} \text{ m}$$

EXAMPLE 4: The pulley is smooth. The system is released from rest with the 1.5 kg mass 3 m above the ground. Find the acceleration of each particle as the 1.5 kg mass is moving towards the ground. Find the time for which the 1.5 kg particle is on the ground.

SOLUTION

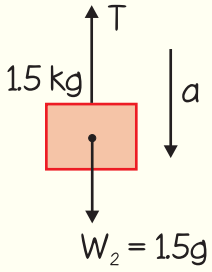
$F_{\text{Net}} = ma$



$T - W_1 = 0.5a$

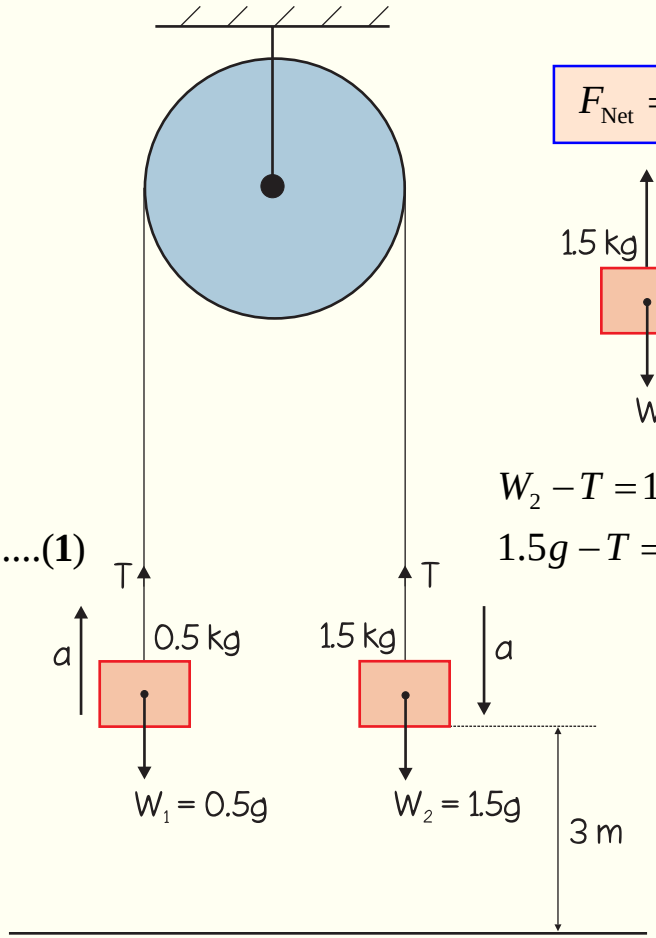
$T - 0.5g = 0.5a.....(\mathbf{1})$

$F_{\text{Net}} = ma$



$W_2 - T = 1.5a$

$1.5g - T = 1.5a.....(\mathbf{2})$



MATHEMATICAL CALCULATIONS

$T - 0.5g = 0.5a....(\mathbf{1})$

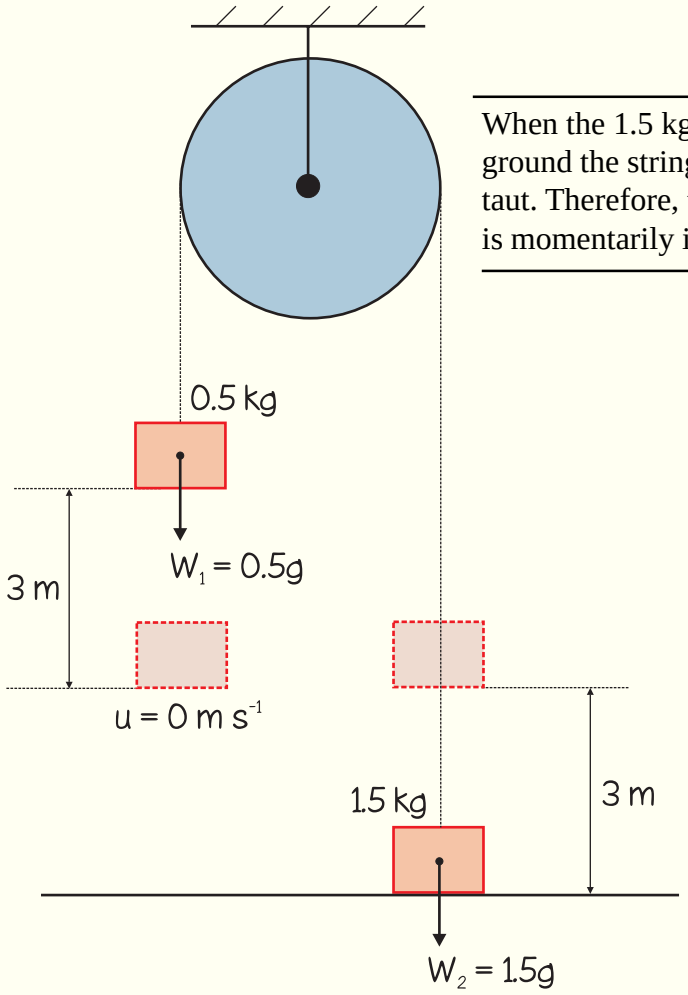
$1.5g - T = 1.5a.....(\mathbf{2})$

$1g = 2a \Rightarrow a = \frac{1}{2}g \text{ ms}^{-2}$

CONT.....

EXAMPLE 4: The pulley is smooth. The system is released from rest with the 1.5 kg mass 3 m above the ground. Find the acceleration of each particle as the 1.5 kg mass is moving towards the ground. Find the time for which the 1.5 kg particle is on the ground.

SOLUTION



When the 1.5 kg mass hits the ground the string is no longer taut. Therefore, the 0.5 kg mass is momentarily in free fall.

MATHEMATICAL CALCULATIONS

$T - 0.5g = 0.5a$....(**1**)

$1.5g - T = 1.5a$.....(**2**)

$1g = 2a \Rightarrow a = \frac{1}{2}g$ ms^{-2}

$u = 0$ ms^{-1}

$s = 3$ m

$a = \frac{1}{3}g$ ms^{-2}

$v = ?$

$v^2 = u^2 + 2as$

$= 0^2 + 2(\frac{1}{3}g)(3)$

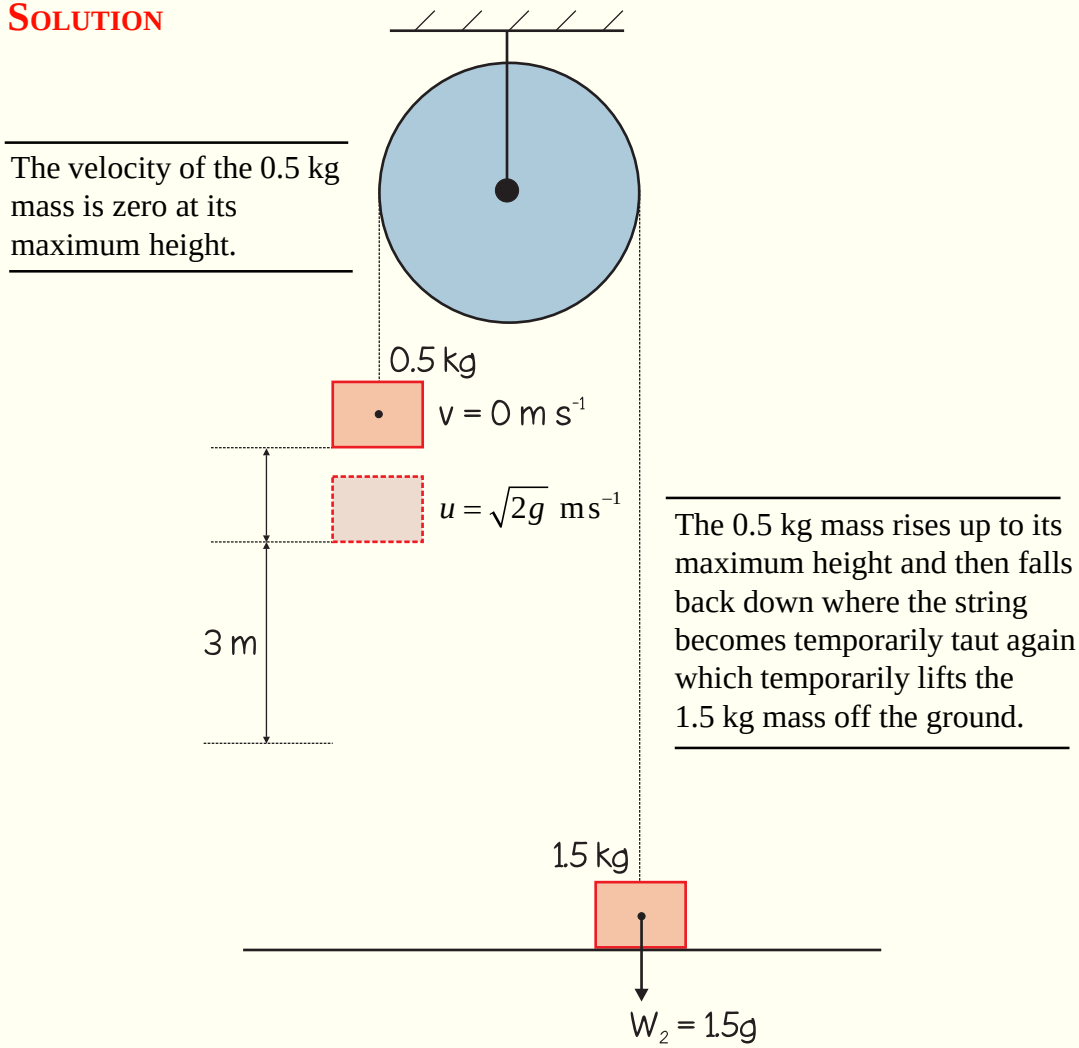
$= 2g$

$v = \sqrt{2g}$ ms^{-1}

CONT.....

EXAMPLE 4: The pulley is smooth. The system is released from rest with the 1.5 kg mass 3 m above the ground. Find the acceleration of each particle as the 1.5 kg mass is moving towards the ground. Find the time for which the 1.5 kg particle is on the ground.

SOLUTION



MATHEMATICAL CALCULATIONS

$T - 0.5g = 0.5a$....(**1**)

$1.5g - T = 1.5a$.....(**2**)

$1g = 2a \Rightarrow a = \frac{1}{2}g$ ms^{-2}

1.5 kg mass hits the ground

$u = 0$ ms^{-1}
$s = 3$ m
$a = \frac{1}{3}g$ ms^{-2}
$v = ?$

$v^2 = u^2 + 2as$
$= 0^2 + 2(\frac{1}{3}g)(3)$
$= 2g$
$v = \sqrt{2g}$ ms^{-1}

$u = \sqrt{2g}$ ms^{-1}
$v = 0$ ms^{-1}
$a = \frac{1}{2}g$ ms^{-2}
$t = ?$

$v = u + at$
$0 = \sqrt{2g} + (-g)t$
$gt = \sqrt{2g}$
$t = \frac{\sqrt{2g}}{g} = \sqrt{\frac{2}{g}}$ s

Time up = Time down

Time on the ground = $2\sqrt{\frac{2}{g}}$ s