

LEAVING CERT QUESTIONS

2008

10. (a) If

$$x^2 y \frac{dy}{dx} + y \frac{dy}{dx} = 1$$

and $y = 0$ when $x = 0$, find the value of x when $y = \sqrt{\frac{\pi}{2}}$.

- (b)** A train of mass 200 tonnes moves along a straight level track against a resistance of $400v^2$, where v m/s is the speed of the train. The engine exerts a constant power of P kW.

The acceleration of the train is $\frac{8000 - v^3}{500v}$.

- (i)** Find the value of P .
(ii) The train travels a distance 69.07 m while its speed increases from 10 m/s to v_1 m/s. Find the value of v_1 .

2007

10. (a) Solve the differential equation

$$\frac{dy}{dx} = y^2 \sin x$$

given that $y = 1$ when $x = \frac{\pi}{2}$.

- (b)** The acceleration of a racing car at a speed of v m/s is

$$\left(1 - \frac{v^2}{3200}\right) \text{ m/s}^2.$$

The car starts from rest.

Calculate correct to two decimal places

- (i)** the speed of the car when it has travelled 1500 m from rest,
(ii) the maximum speed of the car.

2006

10. (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{xy}{1+x}$$

given that $y = e$ when $x = 0$.

(b) The acceleration of a particle moving horizontally in a straight line is $\frac{1}{x^3}$ away from a

fixed point o , where x is its distance from o .

The particle starts from rest at $x = 1$.

Calculate

(i) the velocity of the particle when $x = \frac{4}{3}$,

(ii) the time that it takes the particle to reach a point 2 metres from o .

2005

10. (a) Solve the differential equation

$$x \frac{dy}{dx} - xy - y = 0$$

given that $y = 1$ when $x = 1$.

(b) A mass of 9 kg is suspended at the lower end of a light vertical rope.

Initially the mass is at rest. The mass is pulled up vertically with an initial pull on the rope of 137.2 N.

The pull diminishes uniformly at the rate of 1 N for each metre through which the mass is raised.

(i) Show that the resultant upward force on the mass when it is x metres above its initial position is $49 - x$.

(ii) Find the speed of the mass when it has been raised 15 metres.

(iii) Find the work done by the pull on the rope when the mass has been raised by 15 m.

2004

10. (a) Solve the differential equation

$$x^2 \frac{dy}{dx} - y + 4 = 0$$

given that $y = 5$ when $x = 1$.

(b) A particle is projected vertically upwards with an initial speed of $2g$ m/s in a medium in which there is a resistance kv^2 N per unit mass where v is the speed of the particle and k is a constant, where $k > 0$.

(i) Prove that the maximum height reached is

$$\frac{1}{2k} \ln(1 + 4kg).$$

(ii) If the speed of the particle is g m/s when it has reached half its maximum height, find the value of k .

2003

10. (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{xy}{2x^2 - 3}$$

given that $y = 1$ when $x = \sqrt{2}$.

(b) A car of mass 490 kg moves along a straight level horizontal road against a resistance of $70v$ N, where v m/s is the speed of the car.

The engine exerts a constant power of 63 kW.

(i) Show that the equation of motion is

$$7 \frac{dv}{dt} = \frac{900 - v^2}{v}.$$

(ii) Calculate, correct to two decimal places, the time it takes the car to increase its speed from 10 m/s to 20 m/s.

2002

10. (a) Solve the differential equation

$$\frac{dy}{dx} = e^{x-y}$$

given that $y = \ln 4$ when $x = 0$.

(b) A particle starts from rest and moves in a horizontal line. Its speed v at time t is given by the equation

$$\frac{dv}{dt} = 100 - v.$$

(i) Find the time taken for the speed of the particle to increase from 25 m/s to 75 m/s.

(ii) How far does the particle travel in going from rest to a speed of 75 m/s?

(iii) Determine the limiting speed, v_1 , of the particle (that is, $v \rightarrow v_1$ as $t \rightarrow \infty$).

2001

10. (a) Find $\frac{d}{dx} \left(\frac{y}{x} \right)$.

Hence, or otherwise, solve the differential equation

$$\frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2} = \frac{1}{x}$$

given that $y = 1$ when $x = 1$.

(b) A car of mass m kg is travelling along a level road. The resistance to motion is mkv^2 N, where v m/s is the speed. When the car is travelling at 14 m/s, the engine cuts out. Ten seconds after the engine cuts out, the speed of the car is 7 m/s.

(i) Show that $k = \frac{1}{140}$.

(ii) The car travels a distance of s metres in the first T seconds after the engine cuts out. Show that

$$s = 140 \ln \left(1 + \frac{T}{10} \right).$$

2000

10. (a) If

$$x \frac{dy}{dx} + xy \frac{dy}{dx} - 1 = 0$$

and $y = 2$ when $x = e$, find, correct to two places of decimals, the positive value of y when $x = e^2$.

(b) The deceleration of a particle moving in a straight line with speed v m/s has magnitude $4e^{\frac{1}{2}v}$ m/s². The particle has an initial speed of 6 m/s.

(i) Find the time t_1 for the speed to decrease to 3 m/s.

(ii) Find the time t_2 for the particle to come to rest.

(iii) Deduce that $\frac{t_2 - t_1}{t_1} = \sqrt{e}$.

1999

10. (a) Solve the differential equation

$$\left(\frac{7}{v^2 + 1} \right) \frac{dv}{dx} = \frac{1}{x}$$

given that $v = 0$ when $x = 1$.

(b) The rocket engine of a 12 tonne missile produces a thrust of 180.1 kN. The missile is launched in a vertical direction. The air resistance is v^2 N where v is the speed of the missile.

(i) Find the speed of the missile after 30 seconds.

(ii) Find the percentage error in this speed if air resistance is ignored.

1998

10. (a) If

$$t \frac{dv}{dt} = v - vt$$

and $v = 3$ and $t = 5$, find the value of v when $t = 6$.

(b) A particle moves in a straight line. The initial speed is u and the retardation is kv^3 , where v is the speed at the time t . If s is the distance travelled in time t , prove

(i) $v = \frac{u}{1 + ksu}$

(ii) $t = \frac{ks^2}{2} + \frac{s}{u}$.

1997

10. (a) If

$$\frac{dy}{dx} + x + xy = 0$$

and $y = 2$ when $x = 0$, find, correct to two places of decimals, the value of y when $x = 1$.

(b) A particle moves in a straight line and undergoes a retardation of $0.04v^3$ m/s², where v is its speed.

(i) If the initial speed of the particle is 25 m/s, find its speed when it has travelled a distance of 49 m.

(ii) Find the time for the speed to reduce from 25 m/s to 15 m/s.

1996

10. (a) Solve the differential equation

$$\frac{dy}{dx} = 4y \cos x$$

if $y = e^2$ when $x = \frac{\pi}{6}$.

(b) A particle of mass m is projected vertically upwards with a velocity v of $\sqrt{\frac{2g}{k}}$, the air resistance being kv^2 per unit mass. Prove that

(i) the greatest height reached by the particle is $\frac{\ln 3}{2k}$,

(ii) the velocity of the particle when passing through the point of projection on the way

down is $\sqrt{\frac{2g}{3k}}$.

ANSWERS

LEAVING CERT. QUESTIONS

- 2008** 10. (a) $x = 1$ (b) (i) $P = 3200$ (ii) $v_1 = 15$ m/s
- 2007** 10. (a) $y = \frac{1}{1 + \cos x}$ (b) (i) 44.12 m/s (ii) 56.57 m/s
- 2006** 10. (a) $y = \frac{e^{x+1}}{1+x}$ (b) (i) $\frac{1}{4}\sqrt{7}$ (ii) $\sqrt{3}$
- 2005** 10. (a) $y = xe^{x-1}$ (b) (ii) 11.76 (iii) 622.5
- 2004** 10. (a) $y = e^{-\frac{1}{x}+1} + 4$ (b) (ii) $k = \frac{2}{g}$
- 2003** 10. (a) $y = (2x^2 - 3)^{\frac{1}{4}}$ (b) (ii) 1.65 s
- 2002** 10. (a) $y = \ln(e^x + 3)$ (b) (i) $\ln 3$ (ii) 63.63 m (iii) 100 m/s
- 2001** 10. (a) $y = x(\ln x + 1)$
- 2000** 10. (a) $y = 2.32$ (b) (i) $t_1 = \frac{3}{2}\left(\frac{1}{\sqrt{e}} - \frac{1}{e}\right)$ (ii) $t_2 = \frac{3}{2}\left(1 - \frac{1}{e}\right)$
- 1999** 10. (a) $v = \tan\left(\frac{1}{7}\ln x\right)$ (b) (i) 138.64 m/s (ii) 12.7%
- 1998** 10. (a) $v = \frac{18}{5e}$
- 1997** 10. (a) 0.82 (b) (i) 0.5 m/s (ii) $\frac{8}{225}$ s
- 1996** 10. (a) $y = e^{4\sin x}$