Exercise 5. Power

1. An engine pulls a train along a level track against a resistance equal to K times its momentum. The engine works at a constant power of $25KM$ where M is the total mass of the train and the engine and K is a constant. Show that the equation of motion is given by

$$\frac{dv}{dt} = \frac{K(25 - v^2)}{v}.$$

Find the time to increase its speed from 0 m/s to 4 m/s. [**LC 1973**]

2. **(a)** Solve $2x(1+y)\dfrac{dx}{dy} = 8 + x^2$ if $x = 2$ when $y = 3$.

(b) A train of mass m is moving with constant speed of 16 m/s on a level track. If the resistance is 60 N per tonne find the power of the engines.

It now begins to climb an incline of $\sin^{-1}(\frac{1}{98})$. Assuming the power of the engines and the resistance is the same show that the equation of motion is $\dfrac{v}{v-6}\dfrac{dv}{dt} = -\dfrac{4}{25}$. Find the time taken for the velocity to fall to 12 m/s. [**LC 1987**]

3. The engines of an aircraft are developing power at a constant rate H, and the resistance is proportional to the square of the velocity. If the mass of the aircraft is m and its maximum speed is V prove that the distance travelled is increasing its speed from aV to bV is

$$\left(\frac{mV^3}{3H}\right)\ln\left(\frac{1-a^3}{1-b^3}\right).$$

4. The engine of a car of mass 800 kg works at a constant rate of 32 kW. If the resistance to the motion is proportional to the velocity and the velocity t seconds after starting from rest is v m s^{-1} prove that $800v\dfrac{dv}{dt} = 32000 - kv^2$ where k is a constant. If the maximum speed is 40 m s^{-1} find k and hence prove that $v = 40\sqrt{1 - e^{-\frac{1}{20}t}}$.

5. With its engine working at a constant rate of 10 kW, a car of mass 800 kg accelerates along a level road from rest to a maximum speed $v_{max.}$.
It experiences a resistive force of $20v$. Find
(a) $v_{max.}$,
(b) time taken for the car to increase its speed from 10 m s^{-1} to 20 m s^{-1}.

Exercise 5.
Answers

1. $\dfrac{1}{2K}\ln\left(\dfrac{25}{9}\right)$ s **2.** (a) $\frac{1}{3}(5 + x^2)$ (b)

5. 22.4 m s^{-1}, 27.7 s