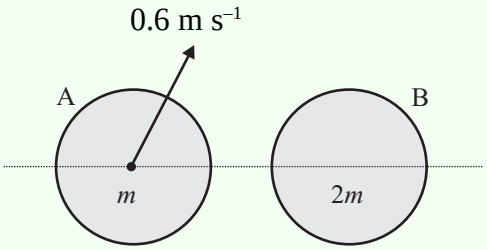
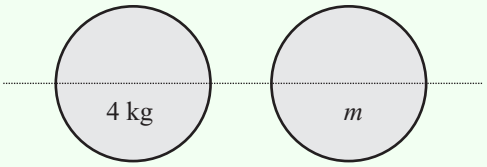
Exercise 7. Oblique Collisions 2

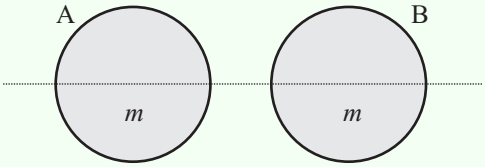
1. A smooth sphere A of mass m moving with speed 0.6 m s^{-1} impinges obliquely on a smooth sphere B of mass $2m$ which is at rest. After collision A is found to move with speed 0.2 m s^{-1} in a direction at right angles to its original direction.
Find the direction of A before impact.
Find the coefficient of restitution.
Show the loss in kinetic energy is $0.06m$ J.



2. A smooth sphere of mass 4 kg collides obliquely with another smooth sphere of mass m which is at rest. After impact the two spheres move at right angles to each other. If $e = \frac{4}{7}$ find m.

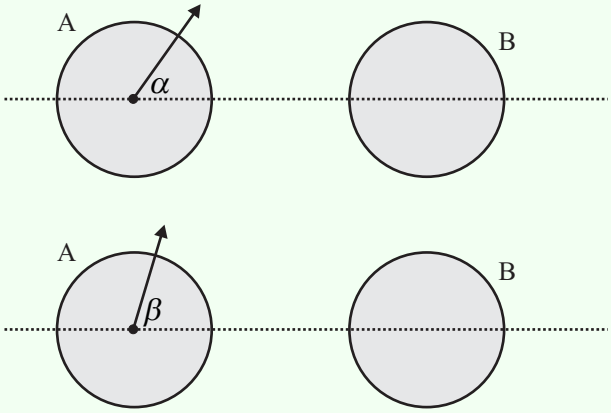


3. A small sphere A of mass m collides obliquely with an identical sphere also of mass m. The spheres are perfectly elastic (e = 1).
Show that the paths of the two spheres after collision are at right angles.
Prove that there is no loss in kinetic energy.

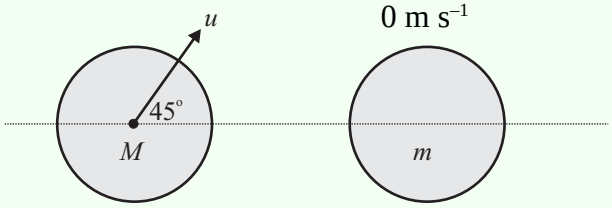


4. A sphere A collides obliquely with another smooth sphere B of equal mass which is at rest.

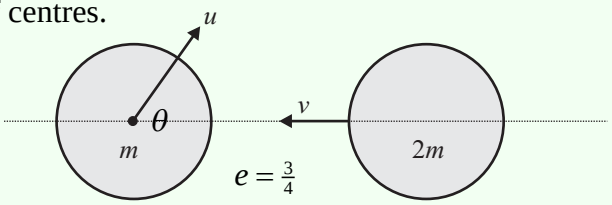
Before impact the direction of motion of A makes an angle α with the line of centres. After impact it makes an angle β with the line of centres. If the coefficient of restitution is $\frac{1}{2}$ show that $\tan\beta = 4\tan\alpha$.



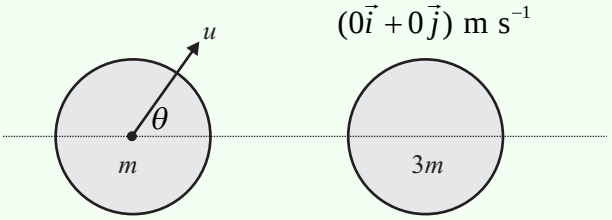
5. After impact the direction of motion of the spheres are at right angles. Show that $m = 2M$.
Find their speeds and directions of motion after. Show that one-quarter of the kinetic energy is lost.



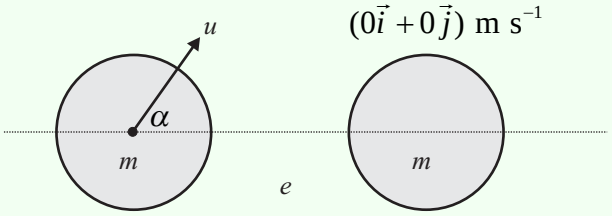
6. The sphere of mass *m* is deflected through an angle of 90°. If $\cos\theta = \frac{3}{7}$ show that $v = \frac{11}{7}u$ at an angle θ to the line of centres.



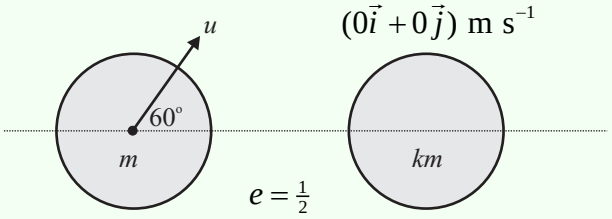
7. A smooth sphere A of mass *m* moving with speed *u* strikes a stationary sphere B of mass 3*m* obliquely. After impact the direction of motion of A is turned through a right angle. Show $\tan^2\theta = \frac{1}{4}(3e-1)$.



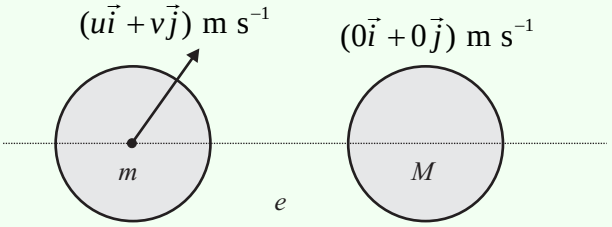
8. A smooth sphere at rest on a smooth horizontal table is struck by an identical sphere which is moving in a direction making an angle α with the line of centres. Find the angle that the first sphere makes with the line of centres after collision where *e* is the coefficient of restitution.



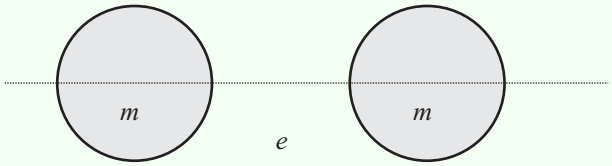
9. Two smooth spheres A and B of the same radius but of masses *m* and *km* respectively rest on a horizontal table. A is projected along the table towards B with speed *u* so as to strike B obliquely at 60° with the line of centres. Show that B's speed after impact is $\frac{3u}{4(k+1)}$ and find A's speed. If after impact A moves in a direction making an acute angle of $\tan^{-1}(2\sqrt{3})$ with B's direction of motion find *k* and show that the loss in kinetic energy is $\frac{1}{32}mu^2$ if $e = \frac{1}{2}$.



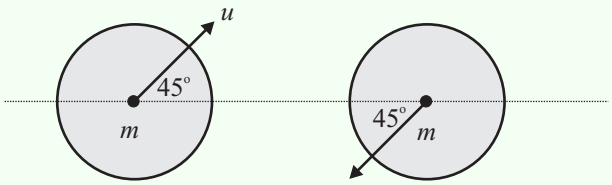
10. A sphere of mass *m* impinges obliquely on a sphere of mass *M* which is at rest. Show that if *m* = *eM* the direction of motion after impact are at right angles where *e* is the coefficient of restitution.



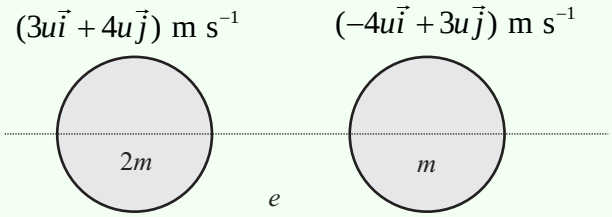
11. A smooth sphere A is moving on a smooth horizontal table. It strikes another identical stationary sphere obliquely. After impact the directions of motion of the two spheres make equal angles θ with the original direction of the motion of A. Prove that $e = \tan^2\theta$ where e is the coefficient of restitution.



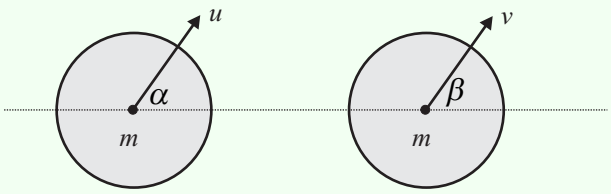
12. Two equal spheres moving with equal speeds in opposite directions impinge obliquely the line of centres on impact being inclined at 45° to the direction of motion of each. Prove that the loss in kinetic energy is half what it would have been had the impact been direct.



13. Two smooth spheres A and B have masses $2m$ and m respectively and velocity vectors $3u\vec{i} + 4u\vec{j}$ and $-4u\vec{i} + 3u\vec{j}$ respectively when they collide with their line of centres parallel to the unit vector $\vec{i}$. If the impact causes a loss of energy equal to the original kinetic energy of sphere B prove that $e = \sqrt{\frac{23}{29}}$.



14. Two identical smooth spheres impinge obliquely with $e = 1$. If α and β are the angles which their initial directions of motions make with their line of centres, and γ and δ the corresponding angles after impact (all angles measured in the same sense), prove that $\tan\alpha \tan\beta = \tan\gamma \tan\delta$.



15. Two identical spheres lie at rest in contact on a smooth horizontal table when they are struck simultaneously by a third sphere moving at right angles to their line of centres along the table. If this last sphere is brought to rest by impact and all spheres are perfectly smooth prove that $e = \frac{2}{3}$. Show that the fractional loss in kinetic energy is $\frac{1}{3}$.

Answers

Exercise 7

1. $\tan^{-1}(\frac{1}{3})$ with the line of centres, $e = \frac{2}{3}$

2. 7 kg

5. $\frac{u}{\sqrt{2}}$ m s^{-1} perpendicular to the line of centres, $\frac{u}{2\sqrt{2}}$ m s^{-1} along the line of centres.

8. $\tan^{-1}\left(\frac{2\tan\alpha}{1-e}\right)$

9. $\frac{u\sqrt{13k^2+20k+16}}{4(k+1)}$ m s^{-1}