LEAVING CERT QUESTIONS

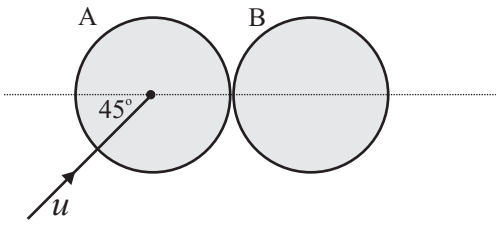
2008

5. (a) Three identical smooth spheres lie at rest on a smooth horizontal table with their centres in a straight line. The first sphere is given a speed 2 m/s and it collides directly with the second sphere. The second sphere then collides directly with the third sphere.
The coefficient of restitution for each collision is e, where $e < 1$.

(i) Find, in terms of e, the speed of each sphere after two collisions have taken place.

(ii) Show that there will be at least one more collision.

(b) A smooth sphere A moving with speed u, collides with an identical smooth sphere B which is at rest.
The direction of motion of A, before impact, makes an angle of 45° with the line of centres at the instant of impact.
The coefficient of restitution between the spheres is e.



Show that the direction of motion of A is deflected through an angle α where

$$\tan\alpha = \frac{1+e}{3-e}.$$

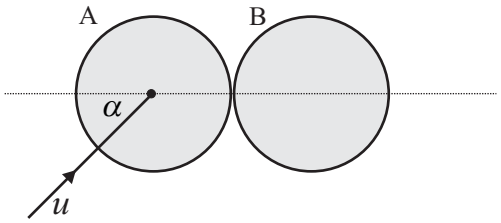
2007

5. (a) A smooth sphere P, of mass 2 kg, moving with speed 9 m/s collides directly with a smooth sphere Q, of mass 3 kg, moving in the same direction with speed 4 m/s.
The coefficient of restitution between the spheres is e.

(i) Find, in terms of e, the speed of each sphere after the collision.

(ii) Show that the magnitude of the momentum transferred from one sphere to the other is $6(1+e)$.

(b) A smooth sphere A, of mass 4 kg, moving with speed u, collides with a stationary smooth sphere B of mass 8 kg.
The direction of motion of A, before impact, makes an angle α with the line of centres at impact.



The coefficient of restitution between the spheres is $\frac{1}{2}$.
Find in terms of u and α

(i) the speed of each sphere after the collision

(ii) the angle through which the 4 kg sphere is deflected as a result of the collision

(iii) the loss in kinetic energy due to the collision.

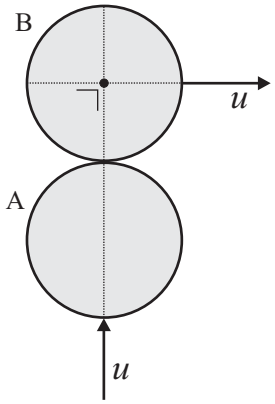
2006

5. (a) A smooth sphere P, of mass 3 kg, moving with speed 6 m/s, collides directly with a smooth sphere Q, of mass 5 kg, which is moving in the same direction with speed 2 m/s. The coefficient of restitution for the collision is e.

(i) Find, in terms of e, the speed of each sphere after the collision.

(ii) If the loss of kinetic energy due to the collision is $k(1-e^2)$, find the value of k.

(b) A smooth sphere A moving with speed u, collides with an identical smooth sphere B which is moving in a perpendicular direction with the same speed u.
The line of centres at the instant of impact is perpendicular to the direction of motion of sphere B.
The coefficient of restitution between the spheres is e.



(i) Find, in terms of e, the speed of each sphere after impact and hence, or otherwise, show that it is not possible for the two spheres to have the same speed after impact.

(ii) Prove that $\tan\theta = \left(\frac{1+e}{2}\right)$, where θ is the angle through which sphere B is turned as a result of the impact.

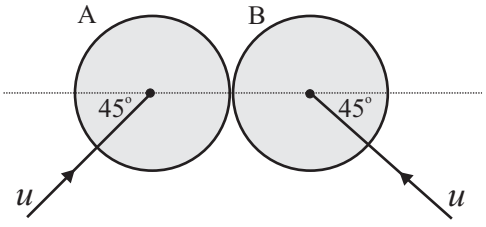
2005

5. (a) Three identical smooth spheres P, Q and R, lie at rest on a smooth horizontal table with their centres in a straight line. Q is between P and R.
Sphere P is projected towards Q with speed 2 m/s. Sphere P collides directly with Q and then Q collides directly with R.

The coefficient of restitution for all of the collisions is $\frac{3}{4}$.

Show that P strikes Q a second time.

(b) A smooth sphere A, of mass m, moving with speed u, collides with an identical smooth sphere B moving with speed u.
The direction of motion of A, before impact, makes an angle 45° with the line of centres at impact.



The direction of motion of B, before impact, makes an angle 45° with the line of centres at impact.
The coefficient of restitution between the spheres is e.

(i) Find, in terms of e and u, the speed of each sphere after the collision.

(ii) If $e = \frac{1}{2}$, show that after the collision the angle between the directions of motion of the two spheres is $\tan^{-1}(\frac{4}{3})$.

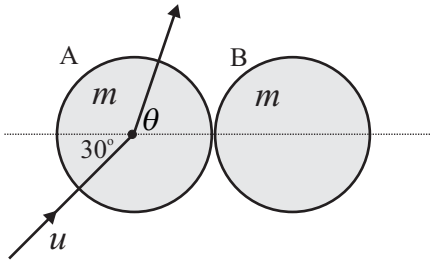
2004

5. (a) A smooth sphere P, of mass $3m$, moving with speed u, collides directly with a smooth sphere Q, of mass $5m$, which is at rest.
The coefficient of restitution for the collision is e.
Find

(i) the speed, in terms of u and e, of each sphere after the collision

(ii) the condition to be satisified by e in order that the spheres move in opposite directions after the collision.

(b) A smooth sphere A, of mass m, moving with speed u, collides with an identical smooth sphere B which is at rest.
The direction of motion of A, before impact, makes an angle 30° with the line of centres at impact.
After impact the direction of A makes an angle θ with the line of centres, where $0^\circ \le \theta < 90^\circ$.
The coefficient of restitution between the spheres is e.
The **speeds** of A and B immediately after impact are equal.

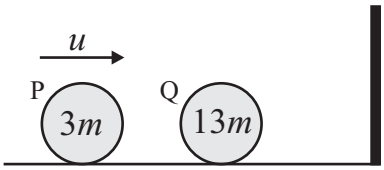


(i) Calculate the value of θ.

(ii) Find e.

2003

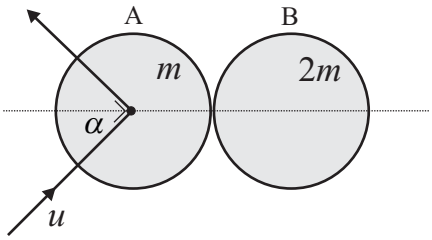
5. (a) A smooth sphere P, of mass $3m$, moving with speed u, collides directly with a smooth sphere Q, of mass $13m$, which is at rest. Sphere Q then collides with a vertical wall which is perpendicular to the direction of motion of the spheres.
The coefficient of restitution for all of the collisions is e.
Find



(i) the speed, in terms of u and e, of each sphere after the first collision

(ii) the range of values of e for which there will be a second collision between the spheres.

(b) A smooth sphere A, of mass m, moving with speed u, collides with a smooth sphere B, of mass $2m$, which is at rest. The direction of motion of A, before impact, makes an angle α with the line of centres at impact, where $0^\circ \le \alpha < 90^\circ$.
As a result of the collision, the direction of A is deflected through an angle of 90°.
The coefficient of restitution between the spheres is e.

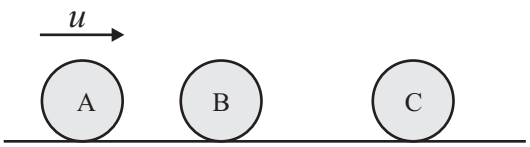


(i) Show that $\tan\alpha = \sqrt{\dfrac{2e-1}{3}}$.

(ii) Find e, if the magnitude of the impulse exerted by A on B is $mu\cos\alpha$.

2002

5. (a) Three identical smooth spheres, A, B and C, lie at rest on a smooth horizontal table with their centres in a straight line. Sphere A is projected towards B with speed u.



Sphere A collides directly with B and then B collides directly with C. Sphere C moves, after the collision, with a speed of $\frac{5}{8}u$.

The coefficient of restitution for each of the two collisions is e.

Find e, correct to two places of decimals.

(b) A smooth sphere P collides with an identical smooth sphere Q which is at rest. The velocity of P before impact makes an angle α with the line of centres at impact, where $0^\circ \le \alpha < 90^\circ$.

The velocity of P is deflected through an angle θ by the collision, so that its velocity after impact makes an angle $\theta + \alpha$ with the line of centres at impact.

The coefficient of restitution between the spheres is $\frac{1}{4}$.

Show that $\tan\theta = \dfrac{5\tan\alpha}{3+8\tan^2\alpha}$.

2001

5. (a) A uniform smooth sphere of mass 2 kg and moving with speed u m/s collides with another uniform smooth sphere of mass 3 kg which is at rest. The velocity of the sphere of mass 2 kg before impact makes an angle of 45° with the line of centres at impact. The coefficient of restitution between the spheres is e.

(i) Find, in terms of e and u, the speed of each sphere after the collision.

(ii) If the sphere of mass 2 kg makes an angle with the line of centres after impact, find

(b) Two identical smooth spheres, each of mass m and moving in the same direction collide directly. The coefficient of restitution between the spheres is e.
If u is the magnitude of the relative velocity between the spheres before impact, show that

(i) each sphere receives an impulse (change in momentum) of magnitude $\frac{1}{2}mu(1+e)$

(ii) the loss in the total kinetic energy of the two spheres due to the impact is $\frac{1}{4}mu^2(1-e^2)$.

2000

5. (a) Two smooth spheres whose masses are m and $2m$ move towards each other in a straight line with speeds $4u$ and u, respectively.

Show that the spheres will move in opposite directions after the collision if $e > \frac{1}{5}$, where e is the coefficient of restitution.

(b) A smooth sphere A collides with an identical smooth sphere B which is at rest. The velocity of A before impact makes an angle α with the line of centres at impact, where $0^\circ \le \alpha < 90^\circ$.

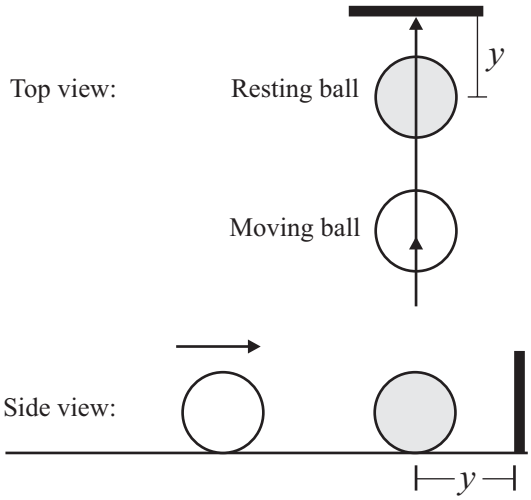
The coefficient of restitution between the spheres is $\frac{1}{2}$.

Show that the angle θ through which the path of A is deflected is given by

$$\tan\theta = \frac{3\tan\alpha}{1+4\tan^2\alpha}.$$

1999

5. (a) A smooth sphere moves on a horizontal table. It strikes an identical sphere at rest on the table. The latter is at a distance y from a vertical cushion. The next collision between the spheres takes place at a distance d from the cushion.



(i) Prove that $d = \dfrac{2e^2 y}{1+e^2}$, where e is the coefficient of restitution for impacts between spheres and between a sphere and the cushion.

(ii) Interpret the result when $e = 1$.

(b) Two equal smooth spheres A and B collide. The velocity of A before the collision is $3\sqrt{3}\,\vec{i} + 3\vec{j}$ and the velocity of B before the collision is $\frac{1}{2}(-u\sqrt{3}\,\vec{i} + u\vec{j})$ where $\vec{i}$ and $\vec{j}$ are unit perpendicular vectors along and perpendicular to the line of centres, respectively.

The velocity of A after the collision is $\frac{1}{2}(-v\vec{i} + v\sqrt{3}\,\vec{j})$.

If the coefficient of restitution is 0.7, find the magnitude and direction of the velocity of sphere B after the collision.

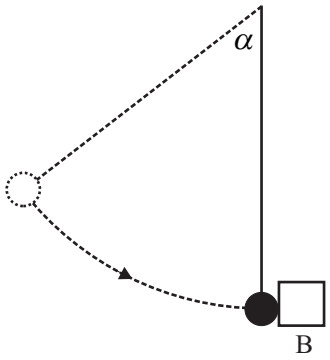
1998

5. (a) Two smooth spheres A and B have masses m_1 and m_2, respectively. They are moving towards each other along the same horizontal line each with speed $2u$. After collision both spheres reverse their original directions of motion and A now travels with speed u.

(i) Show that $3m_1 > 2m_2$.

(ii) Find an expression for e, the coefficient of restitution, and hence or otherwise show that $3m_1 \leq 5m_2$.

(b) A sphere of mass 4 kg is released from rest when $\alpha = 60^\circ$. It swings down and strikes a 7 kg box B when the string is vertical. The distance from the point of support to the centre of the sphere is one metre and the coefficient of restitution for the collision is $\frac{3}{4}$.



Calculate the speed of the box immediately after the impact if the box is free to move.

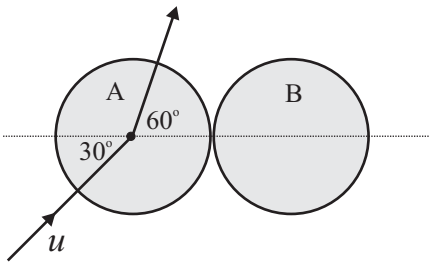
1997

5. (a) A smooth sphere P, of mass m, moving with speed ku collides directly with a smooth sphere Q, of mass km, moving in the same direction with speed u. P is brought to rest by the impact.

(i) Find the velocity of Q after the collision in terms of u.

(ii) Prove that $k \geq 3$.

(b) A smooth sphere A, of mass m, moving with speed u collides with a smooth sphere B, of mass m, which is at rest. The direction of motion of A before impact makes an angle of 30° with the line of centres. If the coefficient of restitution between the sphere is e, find



(i) the velocity of each sphere after impact

(ii) the value of e if after impact the direction of A makes an angle of 60° with the line of centres.

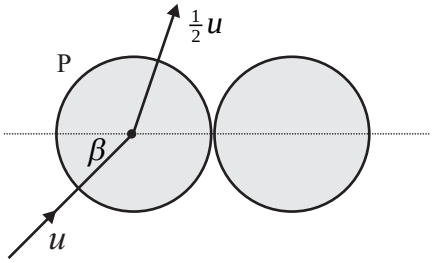
1996

5. (a) Two smooth spheres of mass $2m$ and m moving in opposite directions with speeds u and $2u$, respectively, collide directly. If E_1 and E_2 are the sums of the kinetic energies of the two spheres before and after impact respectively, prove that

$$e = \sqrt{\frac{E_2}{E_1}}$$

where e is the coefficient of restitution.

(b) A smooth sphere P, moving with velocity u, impinges on an equal smooth sphere at rest, the direction of u just before impact being inclined at an angle β to the line of centres.



If the speed of P after impact is $\frac{1}{2}u$ and $\tan\beta = \frac{1}{2}$, show that the coefficient of restitution is also $\frac{1}{2}$.

Answers
Leaving Cert. Questions

2008 5. (a) (i) Speeds after first collision: $(1-e)$ m s^{-1}, $(1+e)$ m s^{-1}

Speeds after second collision: $(1-e)$ m s^{-1}, $\frac{1}{2}(1-e^2)$ m s^{-1}, $\frac{1}{2}(1+e)^2$ m s^{-1}

2007 5. (a) (i) $(6-3e)$ m s^{-1}, $(6+2e)$ m s^{-1}

(b) (i) A: $u\sin\alpha$ m s^{-1}, B: $\frac{1}{2}u\cos\alpha$ m s^{-1} (ii) $90^\circ - \alpha$ (iii) $u^2\cos^2\alpha$ J

2006 5. (a) (i) $\frac{1}{2}(7-5e)$ m s^{-1}, $\frac{1}{2}(7+3e)$ m s^{-1} (ii) $k = 15$

(b) (i) A: $\frac{1}{2}u(1-e)$ m s^{-1}; B: $\frac{1}{2}u\sqrt{e^2+2e+5}$ m s^{-1}

2005 5. (b) (i) A: $\frac{u}{\sqrt{2}}\sqrt{1+e^2}$ m s^{-1}; B: $\frac{u}{\sqrt{2}}\sqrt{1+e^2}$ m s^{-1}

2004 5. (a) (i) $\frac{1}{8}u(3-5e)$ m s^{-1}, $\frac{3}{8}u(1+e)$ m s^{-1} (ii) $\frac{3}{5} < e$

(b) (i) 60° (ii) $e = \frac{1}{3}$

2003 5. (a) (i) $\frac{1}{16}u(3-13e)$ m s^{-1}, $\frac{3}{16}u(1+e)$ m s^{-1} (ii) $0 \le e < \frac{1}{3}$

(b) (ii) $e = \frac{1}{2}$

2002 5. (a) e = 0.58

2001 5. (a) (i) $\frac{u}{5\sqrt{2}}(2-3e)$ m s^{-1}, $\frac{u}{5\sqrt{2}}(2+2e)$ m s^{-1} (ii) $e = \frac{1}{2}, \frac{5}{6}$

1999 5. (a) (ii) Both collisions occur at the same spot.

(b) 4.324 m s^{-1}, $\tan^{-1}\left(\frac{29}{39\sqrt{3}}\right)$

1998 5. (a) (ii) $e = \frac{3m_1 - m_2}{4m_2}$ (b) $\frac{7\sqrt{g}}{11}$ m s^{-1}

1997 5. (a) (i) $2u$

(b) (i) $[\frac{\sqrt{3}}{4}u(1-e)\vec{i} + \frac{1}{2}u\vec{j}]$ m s^{-1}, $[\frac{\sqrt{3}}{4}u(1+e)\vec{i} + 0\vec{j}]$ m s^{-1} (ii) $e = \frac{1}{3}$