Leaving Cert Questions

2008

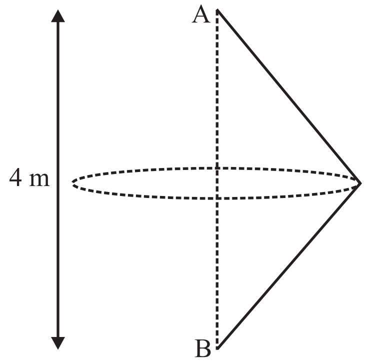
6. (a) A particle of mass 5 kg is suspended from a fixed point by a light elastic string which hangs vertically. The elastic constant of the string is 500 N/m.
The mass is pulled down a vertical distance of 20 cm from the equilibrium position and is then released from rest.

(i) Show that the particle moves with simple harmonic motion.

(ii) Find the speed and acceleration of the mass 0.1 seconds after it is released from rest.

(b) A and B are two fixed pegs, A is 4 m vertically above B.
A mass m kg, connected to A and B by two light inextensible strings of equal length, is describing a horizontal circle with uniform angular velocity ω.

For what value of ω will the tension in the upper string be double the tension in the lower string?



2007

6. (a) A particle of mass m kg is suspended from a fixed point p by a light elastic string.
The extension of the string is d when the particle is in equilibrium.
The particle is then displaced vertically from the equilibrium position a distance not greater than d and is then released from rest.

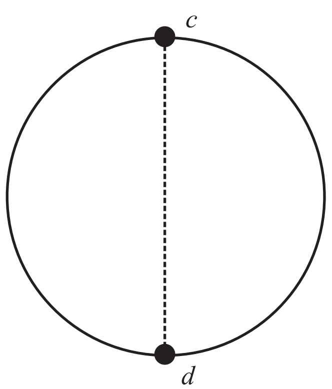
(i) Show that the motion of the particle is simple harmonic.

(ii) Find, in terms of d, the period of the motion.

(b) A bead slides on a smooth fixed circular hoop, of radius r, in a vertical plane.

The bead is projected with speed $\sqrt{10gr}$ from the highest point c.
It impinges upon and coalesces with another bead of equal mass at d.
cd is the vertical diameter of the hoop.

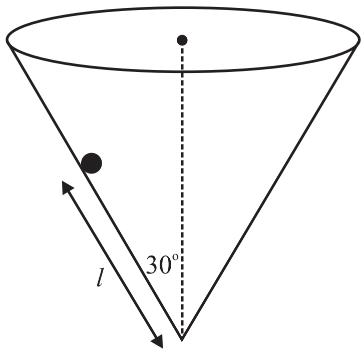
Show that the combined mass will not reach the point c in the subsequent motion.



2006

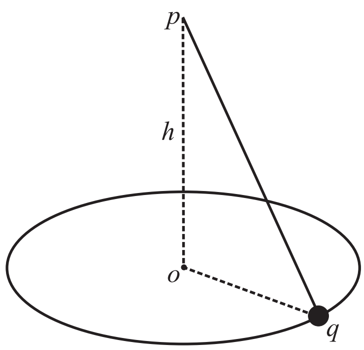
6. **(a)** A particle moves with simple harmonic motion of period 3π. At time $t = 0$, the particle passes through the centre of the oscillation. It passes through a point distant 4 m from the centre of motion with a speed of 5 m/s away from the centre.
Find, correct to two decimal places,
(i) the maximum acceleration of the particle
(ii) the time which elapses before it next passes through this point.

(b) A hollow cone with its vertex downwards and its axis vertical, revolves about its axis with a constant angular velocity of 4π rad/s.
A particle of mass m is placed on the inside rough surface of the cone. The particle remains at rest relative to the cone.
The coefficient of friction between the particle and the cone is $\frac{1}{4}$.
The semi-vertical angle of the cone is 30° and the particle is a distance l m from the vertex of the cone.
Find the maximum value of l, correct to two places of decimals.



2005

6. **(a)** A conical pendulum consists of a light inelastic string $[pq]$, fixed at the end p, with a particle attached to the other end q .
The particle moves uniformly in a horizontal circle whose centre o is vertically below p.
If $|po| = h$, find the period of the motion in terms of h.

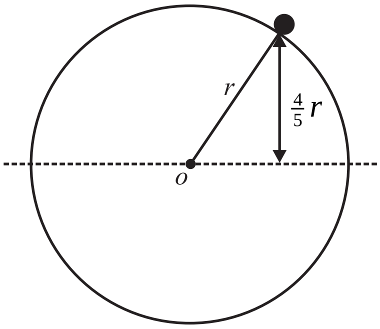


(b) A light elastic string of natural length a and elastic constant k is fixed at one end to a point o on a smooth horizontal table. A particle of mass m is attached to the other end of the string.
Initially the particle is held at rest on the table at a distance $2a$ from o, and is then released.

Show that the time taken for the particle to reach o is $\sqrt{\frac{m}{k}}\left(1+\frac{\pi}{2}\right)$.

2004

6. (a) A particle can move on the smooth outer surface of a fixed sphere of radius r.
The particle is released from rest on the smooth surface of the sphere at a height $\frac{4}{5}r$ above the horizontal plane through the centre o of the sphere.
Find, in terms of r, the height above this plane at which the particle leaves the sphere.



(b) A particle moves in a straight line such that its displacement from a fixed point o at time t is given by

$$x = a\cos(\omega t - \beta)$$

where a, ω and β are positive constants.

(i) Show that the motion of the particle is simple harmonic motion.

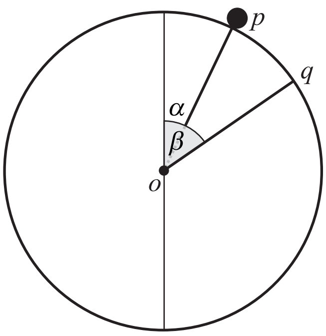
The period of the motion is 16 seconds. At time $t = 4$ s, the particle is 12 m from o and 4 s later the particle is on the other side of o and at a distance of 5 m from o.

(ii) Find a, ω and β.

2003

6. (a) A particle is moving with simple harmonic motion of period π seconds about a fixed point o.
The maximum speed of the particle is 8 cm/s.
(i) Find the amplitude of the motion.
(ii) Find the speed of the particle when it is at a distance of 3 cm from o.

(b) A particle of mass m is held at a point p on the surface of a fixed smooth sphere, centre o and radius r.
op makes an angle α with the upward vertical.
The particle is released from rest.
When the particle reaches an arbitrary point q, its speed is v.
oq makes an angle β with the upward vertical.

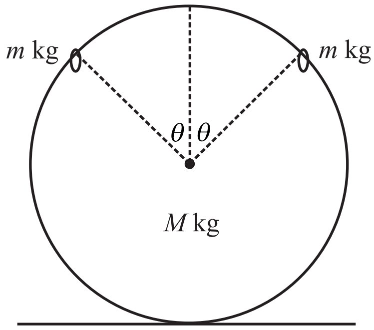


(i) Show that $v^2 = 2gr(\cos\alpha - \cos\beta)$.

(ii) If $\cos\alpha = \frac{2}{3}$ and if q is the point at which the particle leaves the surface, find the value of β.

2002

6. A smooth uniform vertical hoop of radius r and mass M kg stands in a vertical plane on a horizontal surface.
The hoop threads two small rings, each of mass m kg. The rings are released from rest at the top of the hoop.



(i) When the two rings have each fallen through an angle of θ on opposite sides of the hoop, show that the normal force of reaction exerted by the hoop on each ring is

$$mg(3\cos\theta - 2) \text{ N},$$

where this force is taken to act in the outward direction from the centre of the hoop.

(ii) Show that the hoop will rise from the table if $m > \frac{3}{2}M$.

2001

6. (a) A particle moving with simple harmonic motion has speeds of 5 cm/s and 2 cm/s when it is at points 3 cm and 4 cm, respectively, from the centre of the motion.

(i) Find the amplitude and the period of the motion.

(ii) Find the maximum speed of the particle.

(b) A particle of mass m kg is suspended from a fixed point p by a light elastic string of natural length l and elastic constant $\dfrac{4mg}{l}$.

(i) Find the distance of the equilibrium position from the point p, in terms of l.

(ii) The particle is pulled down until it is at a distance $\frac{7}{4}l$ vertically below p and is then released from rest.
Find the time taken, in terms of l, for the string to go slack.

2000

6. **(a)** A particle is placed on a horizontal rotating turntable, 10 cm from the centre of rotation. There is a coefficient of friction of 0.4 between the particle and the turntable. If the speed of the turntable is gradually increased, at what angular speed will the particle begin to slide?

(b) A particle of mass 0.3 kg is attached to the midpoint of a light elastic string of natural length 1 m and elastic constant k. The string is then stretched between two points a and b. The point a is 2 m vertically above b.
Find

(i) the extensions of the two parts of the string, in terms of k, when the system is in equilibrium

(ii) the minimum value of k which will ensure that the lower part of the string is taut

(iii) the period of small oscillations, in terms of k, when the particle is displaced vertically. (Assume both parts of the string remain taut.)

1999

6. **(a)** A particle moves with simple harmonic motion of period $\frac{\pi}{2}$. Initially it is 8 cm from the centre of motion and moving away from the centre with a speed of $4\sqrt{2}$ cm/s. Find an equation for the position of the particle in time t seconds.

(b) A particle of mass 0.5 kg at rest on a smooth horizontal table is attached to two points p and q, which are 1.2 m apart, by two light elastic strings. The string attached to p has a natural length 0.4 m and elastic constant 75 N/m. The string attached to q has a natural length 0.6 m and elastic constant 50 N/m.

(i) Find the equilibrium position.

(ii) Prove that if the particle is displaced in the direction $\overrightarrow{pq}$, through such a distance that neither string goes slack and is then released, it moves with simple harmonic motion.

1998

6. **(a)** Define Simple Harmonic Motion.

The distance, x, of a particle from a fixed point, o, is given by $x = 7\sin\omega t + 24\cos\omega t$, ω being a constant.

(i) Show that the particle is describing simple harmonic motion about o.

(ii) Calculate the amplitude of the motion.

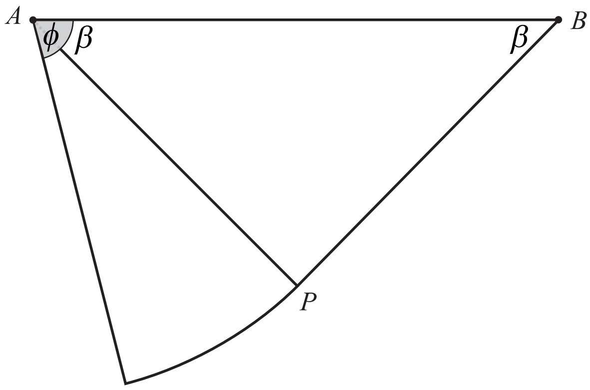
(b) An elastic string of natural length one metre is extended 20 cm by a particle attached to its end and hanging freely. The particle is then pulled down a further distance of 40 cm and released.

(i) Show that the particle moves with simple harmonic motion when the string is taut.

(ii) Find the height above the equilibrium position to which the particle will rise.

1997

6. A particle P, of mass m, is suspended by two inextensible light strings PA, PB, of equal length where A and B are fixed at the same horizontal level and each string is inclined at an angle β to the horizontal.



(i) Find the tension in string PA.

(ii) If the string PB is cut so that P starts to move in a circular path, prove that the tension in the string PA when it makes an angle ϕ with the horizontal is $mg(3\sin\phi - 2\sin\beta)$.

(iii) If the tension in PA is suddenly halved when PB is cut, find the angle β.

1996

6. (a) A body of mass 10 kg moves with simple harmonic motion. At a displacement of 0.8 m from the centre of oscillation, the velocity and acceleration of the body are 2 m/s and 20 m/s^2 respectively.

Find

(i) the number of oscillations per second,

(ii) the amplitude of motion,

(iii) the maximum acceleration and hence show that the force to overcome the inertia of the body at the extremity of the oscillation is 223.6 N.

(b) A light perfectly elastic string of natural length *a* and elastic constant *k* is fastened at one end *p* to a fixed point of a smooth horizontal table, and a particle of mass *m* is attached to the other end. The particle is held on the table at a distance 2*a* from *p* and then released.

Prove

(i) that the particle executes simple harmonic motion while the string is taut,

(ii) that the particle reaches *p* after $\left(\frac{\pi}{2}+1\right)\sqrt{\frac{m}{k}}$ seconds.

Answers
Leaving Cert. Questions

2008 6. (a) (i) SHM about $x = 0$ with $\omega = 10$ (ii) 1.68 m/s, 10.8 m/s^2

(b) $\sqrt{\frac{3g}{2}}$

2007 6. (a) (i) SHM about $x = 0$ with $\omega = \sqrt{\frac{k}{m}}$ (ii) $2\pi\sqrt{\frac{d}{g}}$

2006 6. (a) (i) 3.78 m/s^2 (ii) 3.24 s

(b) 0.43 m

2005 6. (a) $2\pi\sqrt{\frac{h}{g}}$

2004 6. (a) $h = \frac{8}{15}r$

(b) (ii) $a = 13,\ \omega = \frac{1}{8}\pi,\ \beta = \tan^{-1}(\frac{12}{5}) = 1.176$ rad

2003 6. (a) (i) $a = 4$ cm (ii) $v = 2\sqrt{7}$ cm/s

(b) (ii) $\cos^{-1}(\frac{4}{9}) = 63.6^{\circ}$

2001 6. (a) (i) $a = \sqrt{\frac{52}{3}} = 4.16,\ T = \frac{2\pi}{\sqrt{3}} = 3.63$ (ii) $v = 2\sqrt{13}$ cm/s $= 7.2$ cm/s

(b) (i) $\frac{5}{4}l$ (ii) $t = \frac{\pi}{3}\sqrt{\frac{l}{g}}$