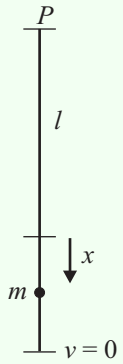
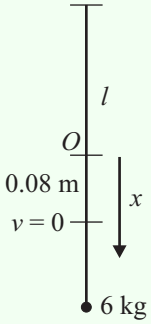
## EXERCISE 11. SIMPLE HARMONIC MOTION 5: VERTICAL ELASTIC STRINGS

**1**. A body of mass $m$ is suspended from a fixed point $p$ by a light inextensible string of natural length $l$ and elastic constant $\frac{49m}{l}$. It is pulled down a distance $\frac{8l}{5}$ and released from rest.



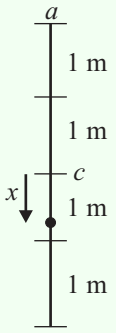

**(i)** Show it performs simple harmonic motion once the string remains taut.
**(ii)** Find in terms of $l$ when the string becomes slack for the first time.

**2**. A mass of 4 kg is suspended by a light spiral spring extends it 8 cm when it is in equilibrium. A second mass of 2 kg is attached to the first and the combined mass is released from rest.

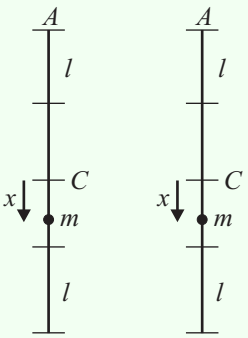



**(i)** Prove its motion is simple harmonic motion.
**(ii)** Find the periodic time.
**(iii)** Find the maximum velocity.

**3**. A particle of mass 2 kg is attached to the ends of two light elastic strings each of natural length 1 m and elastic constant 49 N/m. The other ends of the two strings are attached to two fixed points $a$ and $b$ in the same vertical line where $a$ is 4 m above $b$. The particle is released from rest from the midpoint of $ab$. Show that the particle undergoes simple harmonic motion by considering the forces acting on it when it is $x$ m from the midpoint of $ab$. Find the centre, period and amplitude. Find the time for the particle to reach 0.3 m below the midpoint of $ab$ and the speed at this point.




**4**. A particle $P$ of mass $m$ is attached to the midpoint of a light elastic string of natural length $2l$ and elastic constant $\frac{3mg}{l}$. The string is stretched between two fixed points $A$ and $B$ a distance $3l$ apart with $B$ vertically below $A$. Prove that $P$ lies in equilibrium $\frac{1}{6}l$ below the centre $C$ of $AB$. If $P$ is pulled down $\frac{1}{2}l$ below $C$ and released from rest show it performs simple harmonic motion. Find the centre, amplitude and period of this motion.



**5**. When a particle is hung from a fixed point by a light elastic string it extends it by *l*. If now, it is projected vertically upwards with speed $\sqrt{2gl}$ show that the particle will come to instantaneous rest after a time $\left(\frac{\pi}{4}+1\right)\sqrt{\frac{l}{g}}$. (Assume that the string does not tighten again before the particle reaches this position.)

**6**. When a particle is hung on an elastic thread of natural length *l* it extends it by $\frac{1}{2}l$. If the particle is now pulled down a further distance *l* and released show that it will rise to within $\frac{1}{4}l$ from the point of suspension in time $\sqrt{\frac{2l}{g}}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)$.

**7**. A light elastic spring, of modulus 8*mg* and natural length *l* has one end attached to a ceiling and carries a scale pan of mass *m* at the other end. The scale pan is given a vertical displacement from its equilibrium position and released to oscillate with period *T*. Prove that $T = 2\pi\sqrt{\frac{l}{8g}}$.

**8**. A particle of mass *m* is attached to a point *O* by a light elastic string of unstretched length *l*. It is released from rest at *O* and falls a distance 2*l* before coming to instantaneous rest. If the elastic constant is $\frac{4mg}{l}$ show that the time taken to fall to the lowest point is $\left\{\frac{\pi}{4}+\sqrt{2}+\frac{1}{2}\sin^{-1}\left(\frac{1}{3}\right)\right\}\sqrt{\frac{l}{g}}$. (Can you do it without being given the elastic constant?)

**9**. An elastic string *pq* has natural length 1 m and elastic constant 245 N/m. *P* is fixed and a 5 kg mass is attached at *q*. The particle is held 1 m from *P* and released from rest. Show it moves with simple harmonic motion. Find the centre, the periodic time, the amplitude and the time to fall 0.3 m.

**10**. A body of mass 0.25 kg hangs from a spiral spring. When pulled down 10 cm below its equilibrium position and released from rest. It vibrates with simple harmonic motion of period 2 s.

**(i)** Find its speed as it passes through the equilibrium position.
**(ii)** Find the shortest time to travel from 2 cm below the equilibrium position to 2 cm above the equilibrium position.
**(iii)** Find the elastic constant of the spring.
**(iv)** By how much will the spring shorten when the body is removed?

**11**. One end *O* of a light elastic string, obeying Hooke's law and of natural length *l*, is attached to a fixed point. To the other end *P* of the string is attached a particle of mass *m* which hangs in equilibrium with $OP = \frac{5}{4}l$.

The particle is pulled down vertically a further distance *a* and released from rest.

**(i)** If $a \le \frac{1}{4}$, show that *P* rises a distance 2*a* before first coming to instantaneous rest after a time $\frac{1}{2}\pi\sqrt{\frac{l}{g}}$.

**(ii)** If $\frac{1}{4}l < a < \frac{3}{4}l$, show that *P* rises a distance $\frac{(l+4a)^2}{8l}$ before first coming to instantaneous rest and find its greatest speed during this motion.

**12**. Two particles of equal mass hang on the end of a spring of natural length *l* and whose elastic constant is $\frac{mg}{l}$ where m is the mass of each particle. Find the extension. If one particle is suddenly removed find the time taken for the other particle to reach maximum speed. Find this speed.

**13**. A particle is suspended from a fixed point *O* by means of a light elastic string of natural length *l* and hangs at rest with extension *c*. It is given a downward vertical velocity *v* when in this position. Show that the ensuing motion is simple harmonic once $v^2 \le gc$. If $v^2 = 2gc$ show that the time taken from the lowest point of its path to the highest point is $\sqrt{\frac{c}{g}}\left(1+\frac{3\pi}{4}\right)$.

**14**. One end of a light elastic string of natural length l and elastic constant $\frac{mg}{l}$ is attached to a particle of mass *m*. The other end is attached to a fixed point *O*. The particle is released from rest at a point $\frac{1}{2}l$ below *O*. Prove that it returns to its initial position after a time $\left(\frac{3\pi}{2}+2\right)\sqrt{\frac{l}{g}}$. Prove that the greatest speed attained is $\sqrt{2gl}$.

**EXERCISE 11.**
**ANSWERS**

**1.** (i) Simple harmonic motion at $\frac{6}{5}l$ below $p$, (ii) $\omega = \dfrac{7}{\sqrt{l}}$, $r = \frac{2}{5}l$, $t = \dfrac{2\pi\sqrt{l}}{21}$

**2.** (i) Centre 0.24 m below $O$, (ii) $P = 2\pi\sqrt{\dfrac{3}{25g}}$ (iii) $\dfrac{1}{5}\sqrt{\dfrac{g}{3}}$

**3.** 0.2 m below $c$, $\omega = 7$ rad s$^{-1}$, $r = 0.2$ m, $t = \frac{2}{21}\pi$ s, $v = \frac{7}{10}\sqrt{3}$ m s$^{-1}$

**4.** Centre $\frac{1}{6}l$ below $c$, $P = 2\pi\sqrt{\dfrac{l}{6g}}$, $r = \frac{2}{3}l$

**9.** 0.2 m below $q$, $P = \frac{2}{7}\pi$ s, $r = 0.2$ m, $t = \frac{2}{21}\pi$ s

**10.** (i) $\frac{1}{10}\pi$ m s$^{-1}$ (ii) $\dfrac{2}{\pi}\sin^{-1}\left(\dfrac{1}{5}\right)$ s

(iii) $\frac{1}{4}\pi^2$ (iv) $\dfrac{g}{\pi^2}$ m

**12.** $\frac{1}{4}\pi\sqrt{\dfrac{2l}{g}}$, $\frac{1}{2}\sqrt{2gl}$